Reasoning and Sense Making

Essential components for success on SBAC

Oregon Council of Teachers of Mathematics 2013
Goals for this workshop

- Understand what Reasoning and Sense Making look like in the classroom
- Understand how Reasoning and Sense Making fit into the new SBAC assessments
- Get new ideas on how to include Reasoning and Sense Making in the classroom on a daily basis
How do we define Reasoning and Sense Making?

- **Reasoning** – the process of drawing conclusions on the basis of evidence or stated assumptions

- **Sense Making** – developing understanding of a situation, context or concept by connecting it with existing knowledge

  *(Focus in High School Mathematics: Reasoning and Sense Making. NCTM)*
Reasoning and Sense Making

• When students make the connection that every triangle is half of a rectangle with the same base and height and that is why the area is $\frac{1}{2}$ base times height, they are reasoning.

• When they understand that every formula was developed with reason and can often be proven visually, they are making sense of the situation.
How do we do it?

• Reasoning and Sense Making must become a part of the fabric of the classroom
• We must do it on a daily basis
What are some examples of Reasoning and Sense Making that you already use in your classroom?

Think – Pair - Share
1) Provide tasks that require students to figure things out for themselves.
2) Ask students to restate the problem.
3) Give students time to explore problems appropriately.
4) Support students when they become frustrated instead of giving them the answer.
5) Ask prompting questions.
6) Include wait time after asking a question.
7) Encourage students to ask questions.
8) EXPECT students to justify their reasoning.
9) Share quality work of students with students for reflection.
10) Sustain a classroom climate of respect and safe risk taking.
Mathematical Practices (Common Core)

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning
Mathematical Practices

College instructors rate the [CCSS] Mathematical Practices as being of higher value for students to master in order to succeed in their course than any other CCSS content standards. (Conley, Drummond, Gonzales, Rooseboom, & Stout, 2011)
A problem for you:

A slab of soap on one pan of a scale balances $\frac{3}{4}$ of a slab of soap of equal weight and a $\frac{3}{4}$-pound weight on the other pan. How much does the slab of soap weigh?
This problem is intended to illustrate to students that algebra can be an extension of concrete arithmetic reasoning. Students who solve it without algebra may benefit from the explanations of the students that did use algebra.
Why are Reasoning and Sense Making Important?

• When students connect new learning with their existing knowledge they are more likely to understand and retain the new information. *(Hiebert 2003)*

• To prepare our students to be successful in all areas of their lives.
Students need to be prepared for Smarter Balanced Assessments

• Reasoning and Sense Making are essential components for successful problem solving

• Students need experience solving problems similar to those they will be tested on
Example: Selected Response
No Calculator Problem

Drag the correct number to the exponent of $x$ to rewrite the expression as a single term.

$$\frac{3 \sqrt{x} \cdot x^2 \cdot x^{-\frac{5}{6}}}{\sqrt{x}} = x$$
Example: Constructed Response

A car rental company charges customers an initial charge plus a daily charge to rent cars. The initial charge is $30 and the daily charge is $25.

The rental company charged Jacob $180.

Create an equation that can be used to find the number of days, $x$, Jacob rented the car.

Click the buttons to create your answer.
The value of an antique has increased exponentially, as shown in this graph.

Based on the graph, estimate to the nearest $50 the average rate of change in value of the antique for the following time intervals:

- from 0 to 20 years
- from 20 to 40 years
An equation is shown, where $a$, $b$, and $c$ are integers.

$$y = a|x + b| + c$$

Kyle claims that this equation will always have two roots.

Sandy claims that this equation will always have zero roots.

A. Drag one number into each box to create an equation that supports Kyle’s claim.

B. Drag one number into each box to create an equation that supports Sandy’s claim.

C. Drag one number into each box to create an equation that shows that both Kyle and Sandy are incorrect.
The figure shown is composed of a rectangular prism and half of a sphere. The diameter of the sphere is $x$.

Drag an expression into each box to complete an equation that represents the volume of the figure, $V$, in terms of $x$. 

\[ V = \frac{4}{3} \pi x^3 + \frac{\pi x^3}{12} + \frac{\pi x^3}{6} \]
Assessment Claims for Mathematics

- **Concepts and Procedures**: “Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.”

- **Problem Solving**: “Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.”

- **Communicating Reasoning**: “Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.”

- **Data Analysis and Modeling**: “Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.”
How do we teach Reasoning and Sense Making?

By using questions that lead students through their own thinking out loud.
Guide Students to Develop Formulas From Repeated Reasoning

Find the midpoint between $(2,3)$ and $(-2, -1)$

How could you teach the midpoint formula without bringing up the actual formula?
Find the midpoint

(-2, -1)

(2, 3)
Use patterns to find a formula!

Find the midpoint between.....

• (0,0) and (12,0)           (6,0)
• (2,0) and (10,0)           (6,0)
• (4,0) and (8,0)           (6,0)

• (0,0) and (0,8)            (0,4)
• (0,-2) and (0,10)         (0,4)
• (0,3) and (0,11)         (0,4)

• (3,0) and (7,8)            (5,4)
• (-3,6) and (-5,-4)        (-4,1)
• (-4,-8) and (3,5)        (-1.5,-3.5)
Move to the Abstract......

How can we do these without graphing the exact points?

- (60, 30) and (20, -10)
- (-20, 25) and (0, 35)
- (34, 21) and (11, 15)
- (-13, 45) and (2, -18)
Students summarize.....

- “Add the x coordinates, divide by two”
- “Add the y coordinates, divide by two”
- “Find the numbers halfway between each”

Teacher guides......

- How do we show it with algebra?
- How do we show two different x coordinates? y-coordinates? Let students answer this first, then show them how it is written in the formula
Structured and Successful Struggles: Using Reasoning and Sense Making to produce students who can constructively use strategies developed through collaborative discourse experiences in the classroom to get to deep levels of thinking (the good stuff)
Structured and Successful Struggles!

- Teachers must consistently support and encourage students’ progress toward more sophisticated levels of reasoning

  What is......

- 50% of 12?  1% of 12?  51% of 12?
- 25% of 12?  75% of 12?  76% of 12?
- 76% of 44?  52% of 32?  99% of 16?
MP #5 – Use appropriate tools Strategically

- How much help?
- When do we throw in a question? What type of questions do we ask?
- How much do we let them struggle?
- A student’s misconception is that if I can’t solve the problem in 5 minutes, I won’t be able to. They need to have a bank of familiar resources available and practice using it.
Further than Check for Understanding

- Teach the procedures – teach students what we want by modeling, discussion, example/non-example
- Check for understanding – do they know what you want from them?
- Structure the classroom climate so that everyone is responsible to respond and share
The Invisible Child

• What does a classroom look like when all children are involved in the learning?
• What does a classroom look like when there are some/many students not participating?
• What is your TPOV (Teacher’s Point of View) on how to get all students involved?
Summary - How do we do it?

• Guide students to develop formulas from repeated reasoning
• Use structured and successful struggles
• Build questioning and sharing into the lesson and classroom structure
• Require multiple representations
• Use Smarter Balanced type questions
Use Smarter Balanced Type Tasks

• Do the task
• Stop to analyze what is happening
• Where are students going to be confused – write up questions that will help guide them towards the right path.
• You can even have the students talk about where another student might get stuck – not know what to do. (articulate/verbalize this)
Patterns, Plane and Symbol

• Develop a symbolic representation for a function that produces the number of regions in a plane formed by intersecting lines such that no two lines are parallel and no more than two lines intersect in the same point, as shown in the figure on your handout.

• 1 line, 2 regions
• 2 lines, 4 regions
• 3 lines, 7 regions
What was your symbolic representation?

Method One: Use a recursive formula

\[ R(1) = 2, \quad R(L) = R(L-1) + L \]

or

\[ a_i = 2, \quad a_n = a_{n-1} + n \]
Method Two: Geometric Model
$N(N+1)/2$ for the triangle numbers, then just add one!
\[ \frac{N(N+1)}{2} + 1 \]
1, 3, 6, 10, 15, ……plus one to each
2, 4, 7, 11, 16, ………
Method 3: Technology

Create a scatter plot and find an equation of best fit using the quadratic regression.

\[ y = 0.5x^2 + 0.5x + 1 \]
Method 4: algebraic

\[ y = ax^2 + bx + c \]

Substitute in 3 ordered pairs and solve the system to find

\[ a = \frac{1}{2} \quad b = \frac{1}{2} \quad c = 1 \]
What are your next steps?

1) One thing you can share with another educator soon
2) One thing you can share with your students
3) One thing you would like to learn more about
Resources

- www.octm.org
- Focus in High School Mathematics - Reasoning and Sense Making (NCTM)
- More Good Questions - Great Ways to Differentiate Secondary Mathematics Instruction (Marian Small and Amy Lin)
- www.nctm.org/rsmtasks/
- www.scottkim.com
Other Resources for CCSSM

- http://www.smarterbalanced.org/smarter-balanced-assessments/item
- http://illustrativemathematics.org/
- http://insidemathematics.org/
- http://ode.state.or.us/
- http://www.commoncoreconversation.com