**COSA Common Core State Standards Regional Series**

**“Mathematics in Action”**

A Statewide Regional Series for District and School Leaders of CCSS

# Elementary (3-5) Mathematics Session


## Locations:

April 14, 2014 – Eagle Crest Resort, Redmond, OR
April 17, 2014 – Winston Community Center , Winston, OR
April 28, 2014 – Linn County Expo Center, Albany, OR
April 30, 2014 – Medford, OR
May 6, 2014 - Convention Center, Pendleton , OR

## Mathematics Presenter:

Sarah Schuhl, SMc Curriculum, sarahschuhl@yahoo.com

# CCSSM in Action
## 3 – 5

STUDENT WORK

ASSESSMENT

INSTRUCTION

Sarah Schuhl
SMc Curriculum

## What Do We Expect Students To Learn?

## The CCSS Requires Three Shifts in Mathematics

1. **Focus:** Focus strongly where the standards focus.
2. **Coherence**: *Think* across grades, and *link* to major topics
3. **Rigor:** In major topics, pursue *conceptual* **understanding,** procedural skill and *fluency*, and *application*

## Domains K – 5

| Domain | K | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Counting and Cardinality (CC) | ✓ | | | | | |
| Operations and Algebraic Thinking (OA) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Number and Operations in Base Ten (NBT) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Measurement and Data (MD) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Geometry (G) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Numbers and Operations-Fractions (NF) | | | | ✓ | ✓ | ✓ |

## Why Do We Need the *Mathematical Practices*?

Solve.

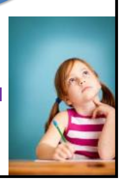
$$\begin{array}{r} 13 \\ -\ 7 \\ \hline \end{array}$$

How did you approach each problem?

$$\frac{5}{10}+\frac{6}{12}=?$$

$$\frac{7}{7}\times\frac{1}{4}\times\frac{3}{3}\times\frac{5}{5}=?$$

How did you think your students will approach each problem?



## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
6. Attend to precision.

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.
5. Use appropriate tools strategically.

7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Essential Question

If the traditional algorithms are not being taught at my grade level….

**What am I supposed to teach?**

## Big Idea: Multiplication & Division (NBT)

Does this make sense?

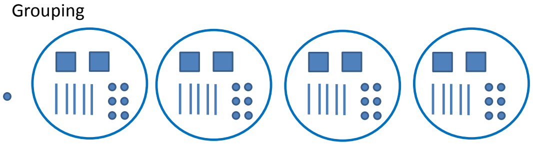
$$\begin{array}{r} 256 \text{ R } 1 \\ 4\overline{)1025} \\ -\underline{8} \\ 22 \\ -\underline{20} \\ 25 \\ -\underline{24} \\ 1 \end{array}$$

As a table group, show as many **strategies** as you can find 1025 ÷ 4. How do you know your strategies makes sense?

Partial Quotients

$$\begin{array}{r|r} 4\overline{)1025} & 100 \\ -\underline{400} & \\ 625 & 100 \\ -\underline{400} & \\ 225 & 50 \\ -\underline{200} & \\ 25 & 6 \\ -\underline{24} & +\underline{\phantom{00}} \\ 1 & 256 \text{ R } 1 \end{array}$$

Grouping



Place Value 1000 ÷ 4 = 250
25 ÷ 4 = 6 R 1

Area Model

| | 100 | 100 | 50 | 6 | ? |
|---|---|---|---|---|---|
| 4 | 400 | 400 | 200 | 24 | 1 |

Compatible Numbers

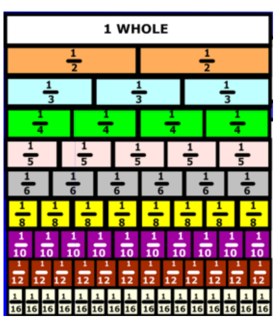
1025 = 400 + 400 + 200 + 24 + 1
400 ÷ 4 = 100
400 ÷ 4 = 100
200 ÷ 4 = 100
24 ÷4 = 6
1 ÷4 = 0.25

- Repeated Subtraction
- Partitioning
- Number Line
- **Others?**

## Big Idea: Fractions

### Unit Fractions

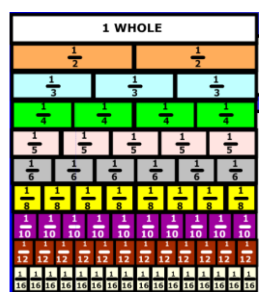
Fill in the Fraction Table with the appropriate unit fractions.




## Unit Fractions

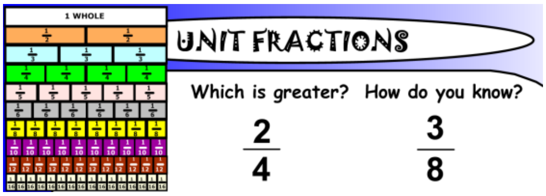
How can you decompose?

$$\frac{3}{4}$$



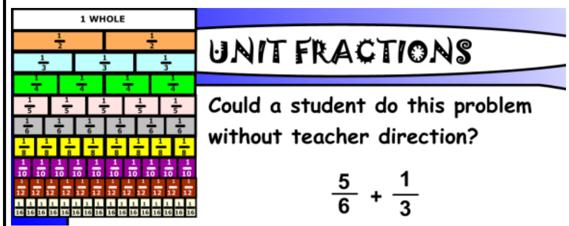

Another possible question:
Name two fractions equivalent to $\frac{3}{4}$.

## Unit Fractions

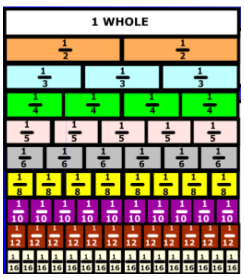



Another possible question:
Show me a number between $\frac{1}{3}$ and $\frac{1}{2}$.

## Unit Fractions




## Unit Fractions




What is $\frac{1}{4} \times \frac{2}{3}$ ?

What is $\frac{4}{5} \div 2$ ?

## Interpreting Division Notation

If I share 5 pizzas among 4 people, how much pizza will each person get? Circle all answers.

$5\overline{)4}$ | 0.8 | $4 \div 5$

1.25 | $\frac{5}{4}$

$5 \div 4$ | $4\overline{)5}$ | $\frac{4}{5}$

## Using High Cognitive Tasks

## Instructional Tasks Matter!

*"Not all tasks are created equal, and* ***different tasks will provoke different levels and kinds of student thinking****."*

Stein, Smith, Henningsen, & Silver, 2000

***"The level and kind of thinking in which students engage determines what they will learn."***

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

## Lower Level Demand Tasks

- Algorithmic.
- Require limited cognitive demand for successful completion. Little ambiguity in problem.
- No connection to concepts/procedures being taught.
- Focused on producing a correct answer instead of developing mathematical understanding.
- Reproduces previously learned facts, rules, formulas, or definitions or requires memorization.

--Smith, M. & Stein, M, *5 Practices for Orchestrating Productive Mathematics Discussions*, 2011 (p. 16)

## Higher Level Demand Tasks

- Focus on using procedures that develop conceptual understanding.
- Often represented in multiple ways.
- Require some cognitive effort. General procedures used cannot be followed mindlessly.
- Require complex and non-algorithmic thinking.
- Require students to explore and understand the nature of math concepts.

--Smith, M. & Stein, M, *5 Practices for Orchestrating Productive Mathematics Discussions*, 2011 (p. 16)

## Five Practices when Implementing High Cognitive Tasks

- **Anticipating** likely student responses to challenging mathematical tasks.
- **Monitoring** students' actual responses to the tasks (while students work on the task in pairs or small groups).
- **Selecting** particular students to present their mathematical work during the whole-class discussion.
- **Sequencing** the student responses that will be displayed in a specific order.
- **Connecting** different students' responses and connecting the responses to key mathematical ideas.

--Smith, M. & Stein, M, *5 Practices for Orchestrating Productive Mathematics Discussions*, 2011 (p. 8)

## Where can I find tasks?

- www.illustrativemathematics.org
- www.k-5mathteachingresources.com
- www.insidemathematics.org
- www.ccssmath.org
- www.commoncoreconversation.com
- www.smarterbalanced.org
- https://grade2commoncoremath.wikispaces.hcpss.org/Grade+2+Home

## Putting it All Together

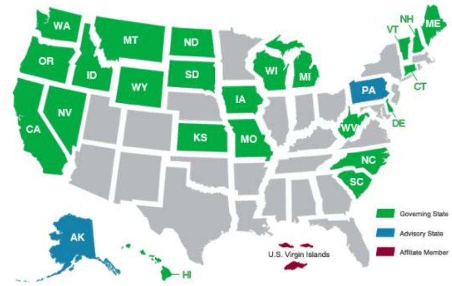
- Design a lesson using the Lesson Planning Tool that you will teach next week.
  - How will you emphasize a mathematical practice?
  - What are your assessing and advancing questions?
  - How will the lesson begin and end?
  - What are students doing during the lesson?
- Find/Create a high cognitive task to use in the next week with students.
  - How will you also teach a mathematical practice?

## How will we know students have learned the CCSSM?

## Formative vs. Summative Assessments

| Formative | Summative |
|---|---|
| A process during learning | An event after learning |
| Descriptive feedback, use of rubrics, student self-assessment | Chapter tests, state assessment, end-of-year placement tests |
| Used to support ongoing growth, improvement | Used to measure achievement |

## SBAC Member States




**SMARTER: Summative Multi-State Assessment Resources for Teachers and Educational Researchers**
**www.smarterbalanced.org**

## Four Claims Used in DRAFT SBAC Test Specifications

**40%**

**Claim #1 Concepts & Procedures** — Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

**Claim #2 Problem Solving** — Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

**Claim #4 Modeling & Data Analysis** — Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

**60%**

**Claim #3 Communicating Reasoning** — Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

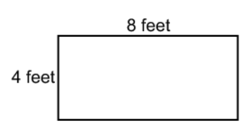
## Assessment Item Types

- **Selected Response (SR)**
  - Variety of multiple choice and true/false
- **Technology Enhanced (TE)**
  - Technology embedded into items
- **Constructed Response (CR)**
  - Free response questions in the Adaptive portion of the test
- **Extended Response (ER)**
  - Non-computer graded constructed response item
- **Performance Tasks (PT)**
  - Rich, real-world scenarios where multiple math topics are addressed

## Cognitive Rigor and Depth of Knowledge (DOK)

- Level 1: **Recall and Reproduction**
  Requires eliciting information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula.
- Level 2: **Basic Skills and Concepts**
  Requires the engagement of some mental processing beyond a recall of information.
- Level 3: **Strategic Thinking and Reasoning**
  Requires reasoning, planning, using evidence, and explanations of thinking.
- Level 4: **Extended Thinking**
  Requires complex reasoning, planning, developing, and thinking most likely over an extended period of time.

## OAKS Question – Grade 4




9
What is the perimeter of this figure?

Ⓐ 12 feet
Ⓑ 20 feet
Ⓒ 24 feet
Ⓓ 32 feet

## SBAC – Grade 4

**43023**

A rectangle is 6 feet long and has a perimeter of $20\frac{1}{3}$ feet.

What is width of this rectangle? Explain how you solved this problem.

## How do you create higher level DOK tasks?

Ask students to:

- Write a word problem for a given expression.
- Write a word problem with a given answer or range of answers.
- Solve a problem using more than one strategy.
- Find the error in a student solution and correct.
- Make sense of a provided solution strategy by writing the original problem or justifying the work shown.
- Solve multi-step problems.
- Solve open-ended tasks with multiple possible responses.

## Does the Assessment Evaluate Student Understanding of Learning Targets?

- Are learning targets clear?
- Do proficient scores indicate student learning?
- Do low scores indicate that students need intervention?

## Is There a Proportional Value Between Scores and Learning Targets on the Assessment?

- Is one learning target weighted more than others?
- Is one assessment method weighted more than another?
- If yes, is that acceptable?

## What Is Proficiency?

- Rubric: Passing in all categories?
- Scoring criteria overall score or each section?
  - PLC team determines.
  - Look at student work.

## Analyze Assessments

- Which standards or learning targets are assessed?
- How are the mathematical practices assessed?
- Use the Evaluation of Assessment Tool to determine balance of DOK Levels, variety of assessment types, quality of questions and final product.
- How will the items be scored?
- What is proficiency?

## Analyze an Assessment

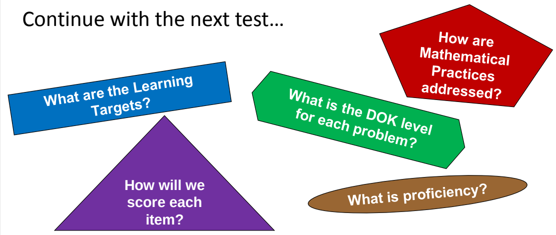
- Look at the assessment.
- How does it measure against the rubric?
- How can it be improved?

**What needs to be modified on your math assessments?**

## Time to create/analyze our tests…

- Choose a current or next unit test
- Analyze or create it using the Evaluation of Assessment Tool
- Discuss any changes that are needed…Continue…

Continue with the next test…




## Analyze Student Work

- Read the task:  Cindy's Cats
- What content standards and/or mathematical practices are being assessed in this task?
- What can you learn from student work?
- What can students learn from one another's work?
- How can *all* students be re-engaged in the learning of this content?

## Next Steps…

- How can you make sure students are learning multiple strategies for conceptual understanding?
- How can you include the standards for mathematical practice in lessons?
- How can you use high cognitive tasks in class?
- What do you need to consider in lesson design?
- What do you need to consider in assessments?

## Contact Information

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## Temperature Check

1. Which mathematical practices have you been teaching students this year?

2. Which content standards have you taught this year?

3. What are three "big ideas" you want students to come to you knowing next year?

4. How are you feeling about implementing the Common Core State Standards in Mathematics?

# Track Your Progress:
# Common Core State Standards for Mathematics in Action

Shade each rectangle to show your current understanding of each learning target.

| | |
|---|---|
| ▪ I can describe strategies for teaching the priority content standards with the mathematical practices. | Starting … Getting There … Got It! |
| ▪ I can create assessments aligned to SBAC claims and DOK levels. | Starting … Getting There … Got It! |
| ▪ I can analyze student work to increase student achievement. | Starting … Getting There … Got It! |

## Next Steps…

# CCSSM (SBAC) <u>Priority</u> Clusters 3 – 5

| Grade 3 | Grade 4 | Grade 5 |
|---|---|---|
| **<u>Operations and Algebraic Thinking</u>**<br>Represent and solve problems involving multiplication and division.<br><br>Understand properties of multiplication and the relationship between multiplication and division.<br><br>Multiply and divide within 100.<br><br>Solve problems involving the four operations, and identify and explain patterns in arithmetic.<br><br>**<u>Number and Operations - Fractions</u>**<br>Develop understanding of fractions as numbers.<br><br>**<u>Measurement and Data</u>**<br>Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects.<br><br>Geometric measurement: understand concepts of area and relate area to multiplication and to addition. | **<u>Operations and Algebraic Thinking</u>**<br>Use the four operations with whole numbers to solve problems.<br><br>**<u>Number and Operations in Base Ten</u>**<br>Generalize place value understanding for multi-digit whole numbers.<br><br>Use place value understanding and properties of operations to perform multi-digit arithmetic.<br><br>**<u>Number and Operations- Fractions</u>**<br>Extend understanding of fraction equivalence and ordering.<br><br>Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.<br><br>Understand decimal notation for fractions, and compare decimal fractions. | **<u>Number and Operations in Base Ten</u>**<br>Understand the place value system.<br><br>Perform operations with multi-digit whole numbers and with decimals to hundredths.<br><br>**<u>Number and Operations -Fractions</u>**<br>Use equivalent fractions as a strategy to add and subtract fractions.<br><br>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.<br><br>**<u>Measurement and Data</u>**<br>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. |

# CCSSM (SBAC) <u>Supporting</u> Clusters 3 – 5

| Grade 3 | Grade 4 | Grade 5 |
|---|---|---|
| **Number and Operations in Base Ten**<br>Use place value understanding and properties of operations to perform multi-digit arithmetic.<br><br>**Measurement and Data**<br>Represent and interpret data.<br><br>Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.<br><br>**Geometry**<br>Reason with shapes and their attributes. | **Operations and Algebraic Thinking**<br>Gain familiarity with factors and multiples.<br><br>Generate and analyze patterns.<br><br>**Measurement and Data**<br>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.<br><br>Represent and interpret data.<br><br>Geometric measurement: understand concepts of angle and measure angles.<br><br>**Geometry**<br>Draw and identify lines and angles, and classify shapes by properties of their lines and angles. | **Operations and Algebraic Thinking**<br>Write and interpret numerical expressions.<br><br>Analyze patterns and relationships.<br><br>**Measurement and Data**<br>Convert like measurement units within a given measurement system.<br><br>Represent and interpret data.<br><br>**Geometry**<br>Graph points on the coordinate plane to solve real-world and mathematical problems.<br><br>Classify two-dimensional figures into categories based on their properties. |

## Essential Skills – CCSSM Content Standards

Review the Priority and Supporting Clusters. Read the accompanying content standards.

My Grade Level: ________

1. What are 7 – 10 Essential Skills students in my grade must learn?

2. What are 7 – 10 Essential Skills students should come to my grade having learned?

# Mathematical Practices 3 – 5

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**Write the number for the mathematical practice best evidenced by each student description.**

| | Student Description | MP |
|---|---|---|
| A | A student is trying to understand what $\frac{3}{8}$ means. When thinking about its relationship to $\frac{1}{2}$, the student thinks about $\frac{3}{8}$ as $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ and that four $\frac{1}{8}$'s is half, so three of them is less than $\frac{1}{2}$. | |
| B | Two students are solving a multi-step word problem. Each student approaches the problem differently. After working together they determine a plan to solve the problem. | |
| C | One student says to find 12 × 3, you can find 12 + 12 + 12. Another explains how to find the product using (10 × 3) + (2 × 3). A third student says to double 6 × 3. The three students discuss which strategy is most efficient. | |
| D | A student writes an equation to represent a word problem and shows how to use it to solve the problem. | |
| E | A student uses his knowledge of decimal operations to figure out the total bill at a restaurant. | |
| F | A student chooses to use a ruler to measure the length of the sides of a rectangle. | |
| G | A student gets an answer of $3\frac{1}{2}$ cars needed and, realizing this is not realistic, changes his answer to 4 cars needed. | |
| H | When looking at a geometric pattern, a student notices each figure has 2 more squares than the previous figure. | |

## Problem Solving Graphic Organizer

<table>
<tr><td><u>Words</u><br><br>Kyle has 12 boxes.<br>There are 6 books in each box.<br>Kyle has room on his shelf for 70 books.<br><br>Will all of the books in the boxes fit on Kyle’s shelf?</td><td><u>Open Number Line</u></td></tr>
<tr><td><u>Area Model</u></td><td><u>Equation</u></td></tr>
<tr><td colspan="2"><u>Answer</u></td></tr>
</table>

# Multiplication & Division: 3 – 5

1. show as many strategies as you can to find 1025 ÷ 4.

2. Use a strategy that is not the standard algorithm to find 678 ÷ 3.

2. Use two different strategies to find 12 × 35.

# Fraction Tables

| WHOLE | EQUIVALENT FRACTION |
|---|---|
| $\frac{1}{2}$ $\frac{1}{2}$ | $\frac{2}{2}$ |
| | |
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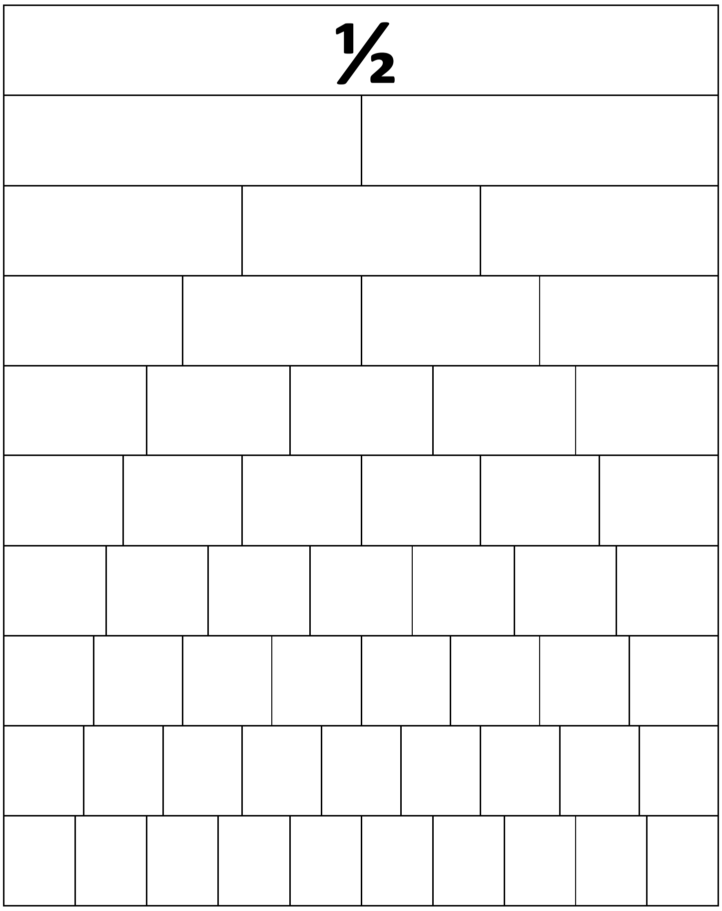
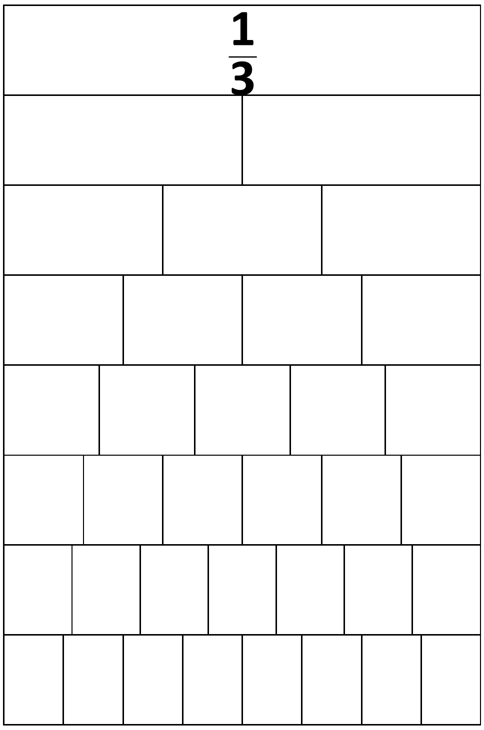



EQUIVALENT FRACTION

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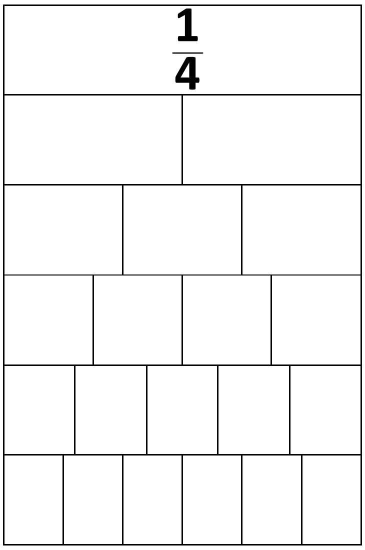


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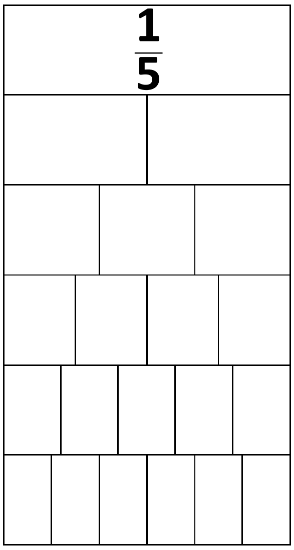
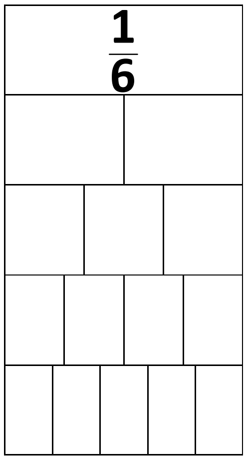



EQUIVALENT FRACTION




EQUIVALENT FRACTION

TABLE 1. Common addition and subtraction situations.[6]

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| **Add to** | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?<br>2 + 3 = ? | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?<br>2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?<br>? + 3 = 5 |
| **Take from** | Five apples were on the table. I ate two apples. How many apples are on the table now?<br>5 – 2 = ? | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?<br>5 – ? = 3 | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?<br>? – 2 = 3 |
| | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown[1]** |
| **Put Together/ Take Apart[2]** | Three red apples and two green apples are on the table. How many apples are on the table?<br>3 + 2 = ? | Five apples are on the table. Three are red and the rest are green. How many apples are green?<br>3 + ? = 5, 5 – 3 = ? | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?<br>5 = 0 + 5, 5 = 5 + 0<br>5 = 1 + 4, 5 = 4 + 1<br>5 = 2 + 3, 5 = 3 + 2 |
| | **Difference Unknown** | **Bigger Unknown** | **Smaller Unknown** |
| **Compare[3]** | ("How many more?" version):<br>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?<br><br>("How many fewer?" version):<br>Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?<br>2 + ? = 5, 5 – 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?<br><br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?<br>2 + 3 = ?, 3 + 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?<br><br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?<br>5 – 3 = ?, ? + 3 = 5 |

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common multiplication and division situations.[7]

| | Unknown Product | Group Size Unknown ("How many in each group?" Division) | Number of Groups Unknown ("How many groups?" Division) |
|---|---|---|---|
| | $3 \times 6 = ?$ | $3 \times ? = 18$, and $18 \div 3 = ?$ | $? \times 6 = 18$, and $18 \div 6 = ?$ |
| **Equal Groups** | There are 3 bags with 6 plums in each bag. How many plums are there in all?<br>*Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?<br>*Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed?<br>*Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| **Arrays,[4] Area[5]** | There are 3 rows of apples with 6 apples in each row. How many apples are there?<br>*Area example*. What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row?<br>*Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?<br>*Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| **Compare** | A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?<br>*Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?<br>*Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?<br>*Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| **General** | $a \times b = ?$ | $a \times ? = p$, and $p \div a = ?$ | $? \times b = p$, and $p \div b = ?$ |

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

## Grade 3
## 3.NBT.3 – Colored Pencils

There are 6 tables in Mrs. Potter's art classroom. There are 4 students sitting at each table. Each student has a box of 10 colored pencils.

(A) How many colored pencils are at each table? Use words, numbers and/or pictures to explain your answer.

(B) Mr. Potter says there are 200 colored pencils all together in his art classroom. Is he correct? Explain your reasoning.

---www.illustrativemathematics.org

# Grade 5
# 5.OA.2 – Video Game

Eric is playing a video game. At a certain point in the game, he has 31500 points. Then the following events happen, in order:

- He earns 2450 additional points.
- He loses 3310 points.
- The game ends, and his score doubles.

1. Write an expression for the number of points Eric has at the end of the game. Do not evaluate the expression. The expression should keep track of what happens in each step listed above.

2. Eric's sister Leila plays the same game. When she is finished playing, her score is given by the expression

$$3(24500+3610)-6780.$$

   Describe a sequence of events that might have led to Leila earning this score.

---www.illustrativemathematics.org

# Figure 2.12:
# CCSS Mathematical Practices Lesson-Planning Tool

| | | |
|---|---|---|
| **Unit:** **Date:** **Lesson:** | | |
| **Learning target:** As a result of today's class, students will be able to ________________ | | |
| **Formative assessment:** How will students be expected to demonstrate mastery of the learning target during in-class checks for understanding? | | |
| Probing Questions for Differentiation on Mathematical Tasks | | |
| **Assessing Questions**<br>(Create questions to scaffold instruction for students who are "stuck" during the lesson or the lesson tasks.) | **Advancing Questions**<br>(Create questions to further learning for students who are ready to advance beyond the learning target.) | |
| **Targeted Standard for Mathematical Practice:**<br>Which Mathematical Practice will be targeted for proficiency development during this lesson? | | |

| **Tasks**<br>(Tasks can vary from lesson to lesson.) | **What Will the Teacher Be Doing?**<br>(How will the teacher present and then monitor student response to the task?) | **What Will the Students Be Doing?**<br>(How will students be actively engaged in each part of the lesson?) |
|---|---|---|
| **Beginning-of-Class Routines**<br>How does the warm-up activity connect to students' prior knowledge, or how is it based on analysis of homework? | | |

| **Tasks**<br>(Tasks can vary from lesson to lesson.) | **What Will the Teacher Be Doing?**<br>(How will the teacher present and then monitor student response to the task?) | **What Will the Students Be Doing?**<br>(How will students be actively engaged in each part of the lesson?) |
|---|---|---|
| **Task 1**<br>How will the students be engaged in understanding the learning target? | | |
| **Task 2**<br>How will the task develop student sense making and reasoning? | | |
| **Task 3**<br>How will the task require student conjectures and communication? | | |
| **Closure**<br>How will student questions and reflections be elicited in the summary of the lesson? How will students' understanding of the learning target be determined? | | |

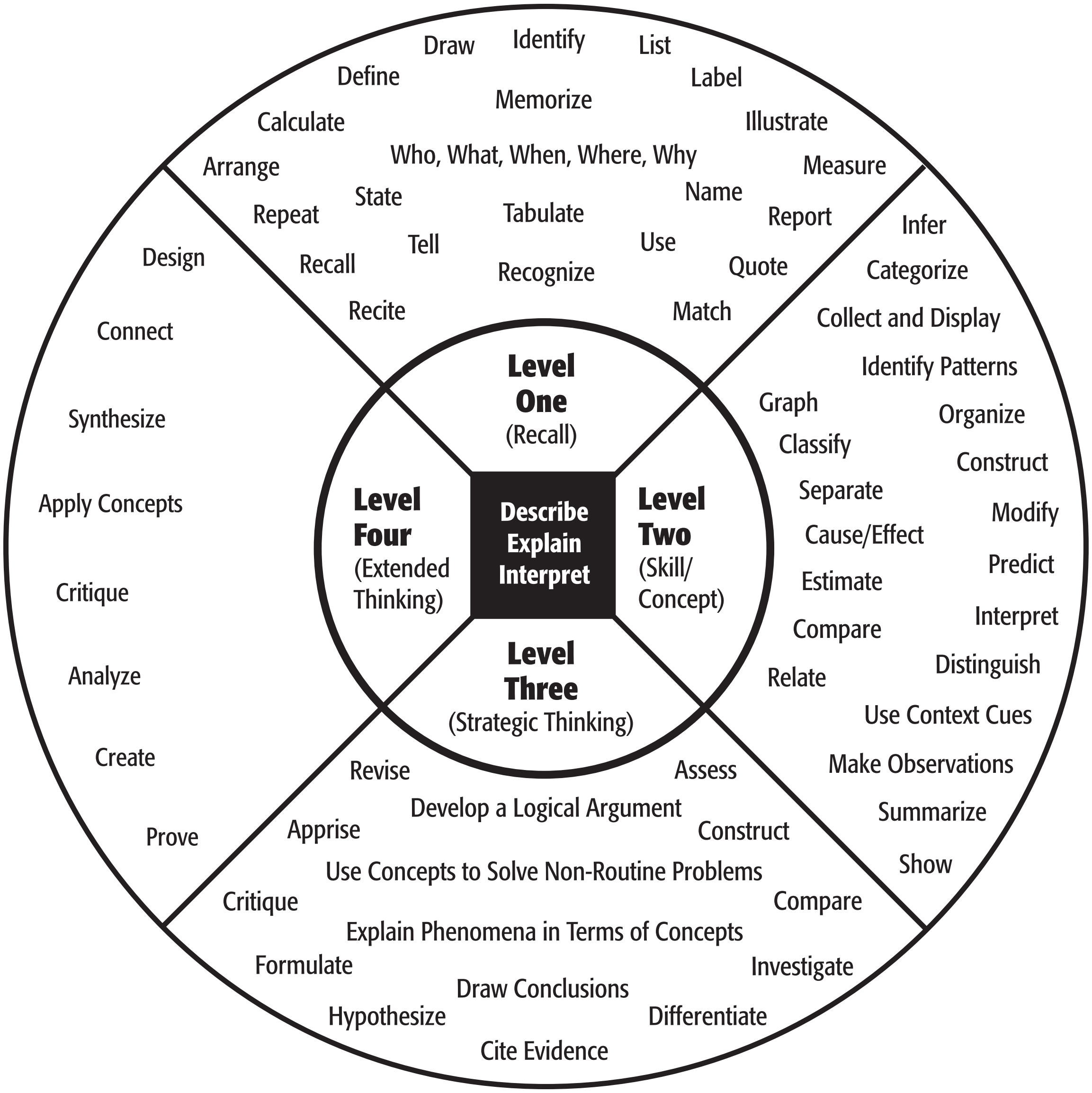
# Depth of Knowledge (DOK)

Source: www.smarterbalanced.org Mathematics Content Specifications

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones & Walkup, 2009)

| **Depth of Thinking (Webb)** + Type of Thinking (Revised Bloom) | **DOK Level 1**<br><br>**Recall & Reproduction** | **DOK Level 2**<br><br>**Basic Skills & Concepts** | **DOK Level 3**<br><br>**Strategic Thinking & Reasoning** | **DOK Level 4**<br><br>**Extended Thinking** |
|---|---|---|---|---|
| Remember | • Recall conversations, terms, facts | | | |
| Understand | • Evaluate an expression<br>• Locate points on a grid or number on number line<br>• Solve a one-step problem<br>• Represent math relationships in words, pictures, or symbols | • Specify, explain relationships<br>• Make basic inferences or logical predictions from data/observations<br>• Use models/diagrams to explain concepts<br>• Make and explain estimates | • Use concepts to solve non-routine problems<br>• Use supporting evidence to justify conjectures, generalize, or connect ideas<br>• Explain reasoning when more than one response is possible<br>• Explain phenomena in terms of concepts | • Relate mathematical concepts to other content areas, other domains<br>• Develop generalizations of the results obtained and the strategies used and apply them to new problem situations |
| Apply | • Follow simple procedures<br>• Calculate, measure, apply a rule (e.g., rounding)<br>• Apply algorithm or formula<br>• Solve linear equations<br>• Make conversions | • Select a procedure and perform it<br>• Solve routine problem applying multiple concepts or decision points<br>• Retrieve information to solve a problem<br>• Translate between representations | • Design investigation for a specific purpose or research question<br>• Use reasoning, planning, and supporting evidence<br>• Translate between problem & symbolic notation when not a direct translation | • Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results |
| Analyze | • Retrieve information from a table or graph to answer a question<br>• Identify a pattern/trend | • Categorize data, figures<br>• Organize, order data<br>• Select appropriate graph and organize & display data<br>• Interpret data from a simple graph<br>• Extend a pattern | • Compare information within or across data sets or texts<br>• Analyze and draw conclusions from data, citing evidence<br>• Generalize a pattern<br>• Interpret data from complex graph | • Analyze multiple sources of evidence or data sets |
| Evaluate | | | • Cite evidence and develop a logical argument<br>• Compare/contrast solution methods<br>• Verify reasonableness | • Apply understanding in a novel way, provide argument or justification for the new application |
| Create | • Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept | • Generate conjectures or hypotheses based on observations or prior knowledge and experience | • Develop an alternative solution<br>• Synthesize information within one data set | • Synthesize information across multiple sources or data sets<br>• Design a model to inform and solve a practical or abstract situation. |

# Depth of Knowledge (DOK) Levels




| Level One Activities | Level Two Activities | Level Three Activities | Level Four Activities |
|---|---|---|---|
| Recall elements and details of story structure, such as sequence of events, character, plot and setting.<br><br>Conduct basic mathematical calculations.<br><br>Label locations on a map.<br><br>Represent in words or diagrams a scientific concept or relationship.<br><br>Perform routine procedures like measuring length or using punctuation marks correctly.<br><br>Describe the features of a place or people. | Identify and summarize the major events in a narrative.<br><br>Use context cues to identify the meaning of unfamiliar words.<br><br>Solve routine multiple-step problems.<br><br>Describe the cause/effect of a particular event.<br><br>Identify patterns in events or behavior.<br><br>Formulate a routine problem given data and conditions.<br><br>Organize, represent and interpret data. | Support ideas with details and examples.<br><br>Use voice appropriate to the purpose and audience.<br><br>Identify research questions and design investigations for a scientific problem.<br><br>Develop a scientific model for a complex situation.<br><br>Determine the author's purpose and describe how it affects the interpretation of a reading selection.<br><br>Apply a concept in other contexts. | Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions.<br><br>Apply mathematical model to illuminate a problem or situation.<br><br>Analyze and synthesize information from multiple sources.<br><br>Describe and illustrate how common themes are found across texts from different cultures.<br><br>Design a mathematical model to inform and solve a practical or abstract situation. |

*Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research, University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>.*

# Figure 4.4: Evaluation Tool for Assessment Instrument Quality

| Assessment indicators | Description of Level 1 | Requirements of the Indicator Are Not Present | Limited Requirements of This Indicator Are Present | Substantially Meets the Requirements of the Indicator | Fully Achieves the Requirements of the Indicator | Description of Level 4 |
|---|---|---|---|---|---|---|
| Identification and emphasis on learning targets | Learning targets are unclear or absent from the assessment instrument. Too much attention is given to one target. | 1 | 2 | 3 | 4 | Clearly stated learning targets are on the assessment and connected to the assessment questions. |
| Visual presentation | Assessment is sloppy, disorganized, and difficult to read. There is no room for teacher feedback. | 1 | 2 | 3 | 4 | Assessment is neat, organized, easy to read, and well spaced. There is room for teacher feedback. |
| Time allotment | Few students can complete the assessment in the time allowed. | 1 | 2 | 3 | 4 | Test can be successfully completed in time allowed. |
| Clarity of directions | Directions are missing or unclear. | 1 | 2 | 3 | 4 | Directions are appropriate and clear. |
| Clear and appropriate scoring rubrics | Scoring rubric is either not in evidence or not appropriate for the assessment task. | 1 | 2 | 3 | 4 | Scoring rubric is clearly stated and appropriate for each problem. |
| Variety of assessment task formats | Assessment contains only one type of questioning strategy and no multiple choice. Calculator usage is not clear. | 1 | 2 | 3 | 4 | Test includes a variety of question types, assesses different formats, and includes calculator usage. |
| Question phrasing (precision) | Wording is vague or misleading. Vocabulary and precision of language is problematic for student understanding. | 1 | 2 | 3 | 4 | Vocabulary is direct, fair, and clearly understood. Students are expected to attend to precision in responses. |
| Balance of procedural fluency and demonstration of understanding | Test is not balanced for rigor. Emphasis is on procedural knowledge. Minimal cognitive demand for demonstration of understanding is present. | 1 | 2 | 3 | 4 | Test is balanced with product- and process-level questions. Higher-cognitive-demand and understanding tasks are present. |

# What does a Common Core Assessment look like?

## Depth of Knowledge Levels

**Level 1: Recall and Reproduce (25% of seat time on assessment)**

**Level 2: Basic Skills and Concepts (50% of seat time on assessment)**

**Level 3: Strategic Thinking and Reasoning (25% of seat time on assessment)**

**Level 4: Extended Thinking (Separate assessment – performance task)**

## Claims

1. **Concepts and Procedures (40% of overall score on SBAC)**
2. **Problem-Solving (40% of overall score on SBAC)**
3. **Communicating Reasoning (20% of overall score on SBAC)**

## Styles of Items

1. Selected Response
   - multiple choice
   - select all that apply
   - true/false or yes/no
   - drag and drop
2. Constructed Response
   - fill in the blank
   - numerical answer
3. Extended Response
   - explain your reasoning
   - show how you know your answer is correct
   - writing a note to convince someone
4. Performance Task

## Assessment Evaluation Tool

| Item Number | DOK Level | Claim | Item Type |
|---|---|---|---|
| | | | |
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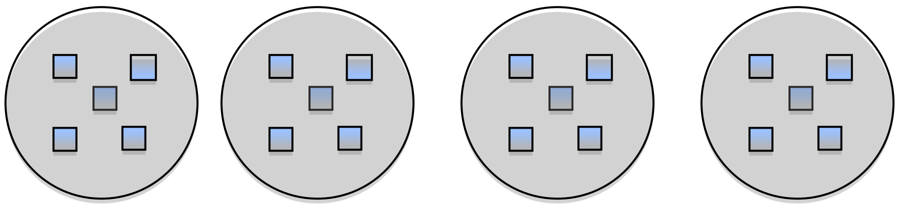
## Grade 3 Unit 4: Multiplication and Division

Name______________________________ Date_______________________

**3.OA.1 – 2** *I can interpret multiplication and division equations.*

1. Destiny found a math picture on the floor after math. Here it is.



a. What are different equations that this picture could represent? Circle all that apply. 2 points______

$4 \times 5 = 20$ $20 \div 4 = 5$ $5 \times 2 = 10$

$10 \div 5 = 2$ $5 \times 4 = 20$ $20 \div 2 = 10$

b. Write a story problem that matches this picture. 1 point ______

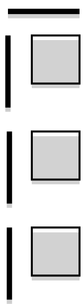
2. Show how you can find the product of $5 \times 6$ on a hundreds chart. 2 points _____

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Answer: $5 \times 6 =$ ________

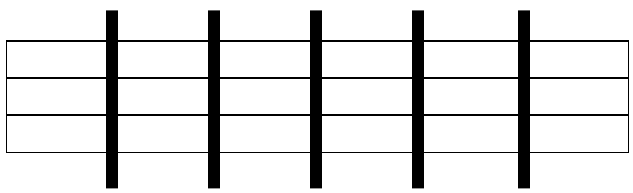
3. Brian's teacher gave him this math problem: $3 \times \square = 21$ 2 points ____

a. Brian started solving the problem by sketching an array. Finish his work to show how he solved the problem.



b. Now, solve the equation $3 \times \square = 21$ $\square =$ ____

4. The model below shows a division problem.



a. Write a division equation that matches the picture. 2 points ______

b. Write a story problem that matches your equation. 1 point ______

**3.OA.3** *I can solve word problems.*

5. Kendra had 8 times as many pencils as Tracy. Kendra has 56 pencils. How many pencils does Tracy have? Use a picture, words, and/or an equation to show how you know your answer is correct. 2 points ______

6. Peggy has 6 pieces of string. Each piece has a length of 8 inches. Peggy placed all six pieces of string in a long line with the end of one string touching the end of another string. What was the total length of the line she created? Use a picture, words, and/or an equation to show how you know your answer is correct. 2 points ______

7. Martin had 4 goldfish bowls. This is what he did with the fish.

He put:

10 fish in the $1^{st}$ bowl
10 fish in the $2^{nd}$ bowl
7 fish in the $3^{rd}$ bowl
and 13 fish in the $4^{th}$ bowl

Martin says that he showed $4 \times 10$ using the bowls and the goldfish. Is he correct? Explain why or why not. 2 points _______

**3.OA.4** *I can determine the unknown in an equation.*

**Determine the unknown number for each problem.** 1 point each ________

8. $6 \times ? = 42$ $? =$ ______

9. $\Delta \times 10 = 70$ $\Delta =$ ______

10. $7 \times 2 = \square$ $\square =$ ______

11. $* \times 9 = 0$ $* =$ ______

12. $36 \div 9 = ?$ $? =$ ______

13. $24 \div \square = 8$ $\square =$ ______

## Scoring

**3.OA.1-2** I can interpret multiplication and division equations.

Test Question #1a: 2 points if all three correct equations are circled
1 point 1 or 2 correct equations are circled
#1b: 1 point for a correct word problem
Test Question #2: 1 point for correct model
1 point for correct answer
Test Question #3: 1 point for correct model in part a
1 point for correct answer in part b
Test Question #4a: 2 points for correct equation
1 point if dividend and divisor correct but wrong quotient
#4b: 1 point for a correct word problem

*Proficiency: 7 out of 10 points

**3.OA.3** I can solve word problems.

Test Question #5: 1 point the picture or equation
1 point for correct answer
Test Question #6: 1 point the picture or equation
1 point for correct answer
Test Question #7: 1 point for answering “No”
1 point for correct explanation

*Proficiency: 4 out of 6 points

**3.OA.4** I can determine the unknown in an equation.

Test Questions #8 – 13 1 point each

*Proficiency: 4 out of 6 points

## Student Reflection

| Learning Target | Test Questions | Score | How did I do? (Circle one.) |
|---|---|---|---|
| **2.NBT.2** I can interpret multiplication and division equations. | #1 – 4 | _____ out of 10 | I got it! Still learning it… |
| **3.OA.3** I can solve word problems. | #5 – 7 | _____ out of 6 | I got it! Still learning it… |
| **3.OA.4** I can determine the unknown in an equation. | #8 – 13 | _____ out of 6 | I got it! Still learning it… |

Learning Targets I know and can do:

Learning Targets I still need to learn:

# Cindy's Cats

This problem gives you the chance to:
• solve fraction problems in a practical context

Cindy has 3 cats: Sammy, Tommy and Suzi.

1. Cindy feeds them on Cat Crunchies.
   Each day Sammy eats $\frac{1}{2}$ of the box, Tommy eats $\frac{1}{8}$ of the box and Suzi eats $\frac{1}{4}$ of the box.
   What fraction of a whole box do the cats eat, in all, each day? ___________
   Show how you figured this out.

2. Tommy and Suzi spend much of each day sleeping.
   Tommy sleeps for $\frac{3}{5}$ of the day and Suzi sleeps for $\frac{7}{10}$ of the day.
   Which of the two cats sleeps for longer? _________________

   How much longer does it sleep each day? _________________
   Show how you figured this out.

3. Cindy's cats often share a carton of cat milk.

Sammy always drinks $\frac{1}{3}$ of the carton, Tommy always drinks $\frac{5}{12}$ of the carton, and Suzi always drinks $\frac{1}{6}$ of the carton.

What fraction of the carton of cat milk is left over? ________________

Show how you figured it out.

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.

Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.

How many times did Suzi use the cat door? ________________

Explain how you figured it out.

______________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

**8**

## Looking at Student Work on Cindy's Cats

An important part of solving word problems is understanding what you know and what is the result of each calculation. Student A makes use of clear labels to define the numbers in the problem and show why the calculations make sense. The student uses common denominators throughout. Student A is able to think about multiplication to find 1/4 of 100 and 3/10 of 100.

**Student A**

1. Cindy feeds them on Cat Crunchies.

   Each day Sammy eats $\frac{1}{2}$ of the box, Tommy eats $\frac{1}{8}$ of the box and Suzi eats $\frac{1}{4}$ of the box.

   What fraction of a whole box do the cats eat, in all, each day? $\frac{7}{8}$

   Show how you figured this out.

   I converted $\frac{1}{2}$ into $\frac{4}{8}$, (for Sammy) then I added $\frac{1}{8}$ (for Tommy), and I converted $\frac{1}{4}$ into $\frac{2}{8}$, (for Suzi) so I did $\frac{4}{8}+\frac{1}{8}+\frac{2}{8}=\frac{7}{8}$ total.

2. Tommy and Suzi spend much of each day sleeping.

   Tommy sleeps for $\frac{3}{5}$ of the day and Suzi sleeps for $\frac{7}{10}$ of the day.

   Which of the two cats sleeps for longer? Suzi

   How much longer does it sleep each day? $\frac{1}{10}$ of the day

   Show how you figured this out.

   I converted $\frac{3}{5}$ into $\frac{6}{10}$, and found out that Suzi sleeps for $\frac{7}{10}$ of the day which is $\frac{1}{10}$ more than what Tommy sleeps.

**Student A, part 2**

3. Cindy's cats often share a carton of cat milk.

Sammy always drinks $\frac{1}{3}$ of the carton, Tommy always drinks $\frac{5}{12}$ of the carton, and Suzi always drinks $\frac{1}{6}$ of the carton.

What fraction of the carton of cat milk is left over? $\frac{1}{12}$ ✓ 1 1

Show how you figured it out.

I converted $\frac{1}{3}$ into $\frac{4}{12}$ (for Sammy), plus $\frac{5}{12}$, (for Tommy) and I also converted $\frac{1}{6}$ into $\frac{2}{12}$, (for Suzi) then I added $\frac{4}{12}+\frac{5}{12}+\frac{2}{12}=\frac{11}{12}$, then I did $\frac{12}{12}-\frac{11}{12}=\frac{1}{12}$ leftover. ✓ 1 1

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.

Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.

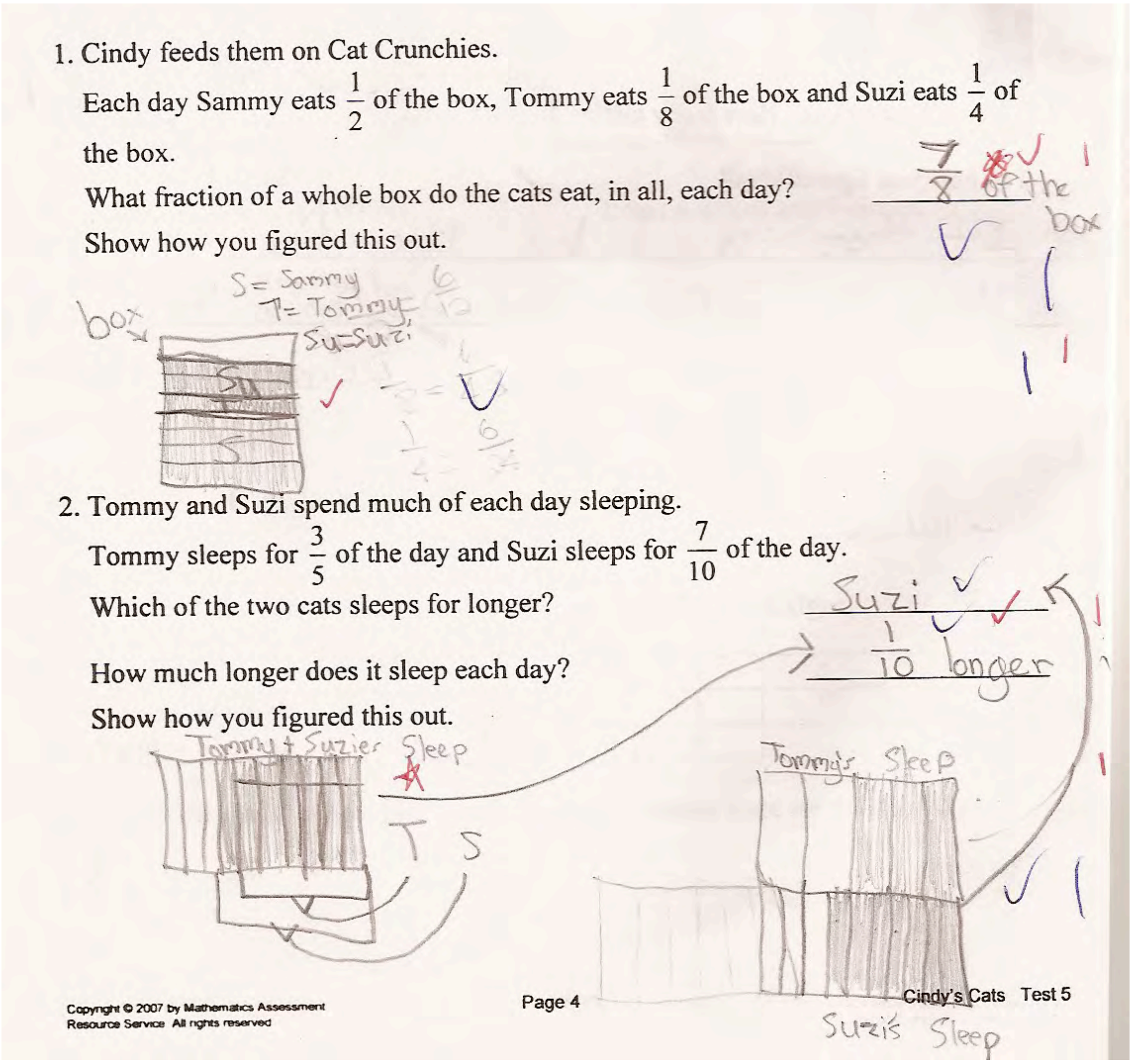
How many times did Suzi use the cat door? 45 times ✓ 1 1

Explain how you figured it out.

$\frac{1}{4}$ of 100 is 25 (for Sammy), $\frac{3}{10}$ of 100 is 30, (for Tommy) then I added 25+30=55, then I did 100−55= 45 jumps, (for Suzi). ✓ 1 1

Student C is able to use diagrams to make sense of the size of the fractions and show the action or operation of the problems. In part 2, Student C shows the comparison in two different ways. *Can you describe the mathematics in each diagram?*

**Student C**



1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats $\frac{1}{2}$ of the box, Tommy eats $\frac{1}{8}$ of the box and Suzi eats $\frac{1}{4}$ of the box.

What fraction of a whole box do the cats eat, in all, each day? $\frac{7}{8}$ of the box

Show how you figured this out.



2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for $\frac{3}{5}$ of the day and Suzi sleeps for $\frac{7}{10}$ of the day.

Which of the two cats sleeps for longer? Suzi

How much longer does it sleep each day? $\frac{1}{10}$ longer

Show how you figured this out.





Page 4 Cindy's Cats Test 5

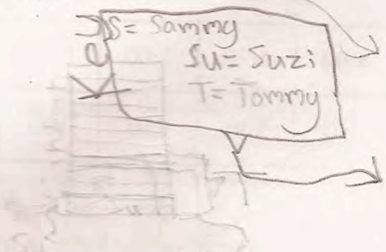
**Student C, part 2**

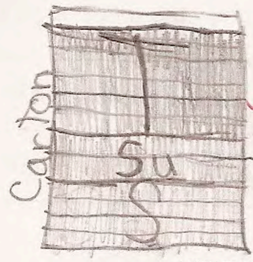
3. Cindy's cats often share a carton of cat milk.

Sammy always drinks $\frac{1}{3}$ of the carton, Tommy always drinks $\frac{5}{12}$ of the carton, and Suzi always drinks $\frac{1}{6}$ of the carton.

What fraction of the carton of cat milk is left over? $\frac{1}{12}$ leftover

Show how you figured it out.




4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.

Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.

How many times did Suzi use the cat door? $\frac{45}{100}$ (45 times)

Explain how you figured it out.

First, I made a graph and figured out how many times Sammy and Tommy used the door. Then the leftover part of the graph I figured must be the amount Suzi used the door.

Student D uses percents to think about the situation in part 4.

**Student D**

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.

Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.

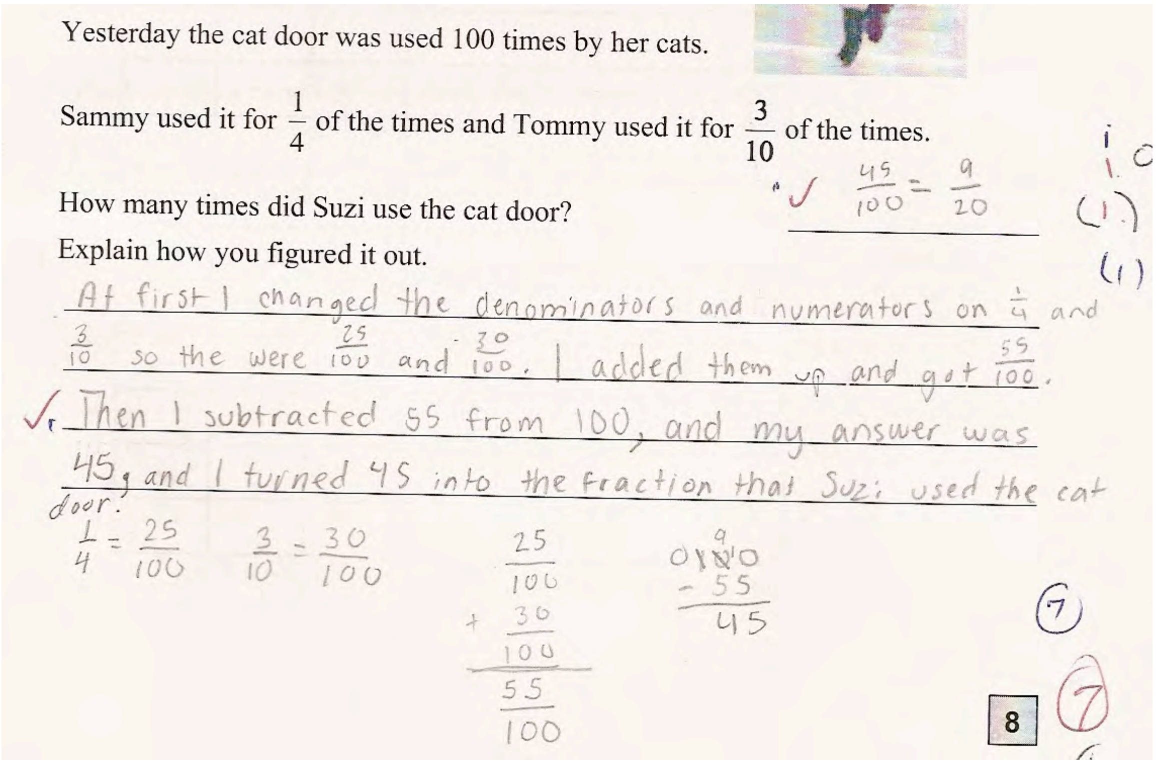
How many times did Suzi use the cat door? 45 times

Explain how you figured it out.

$\frac{1}{4} = 25\%$ $\frac{3}{10} = 30\%$ $30 + 25 = 55$ $100 - 55 = 45$

In part 4, students had a difficult time interpreting their answers. Student E has done all the correct calculations, but can’t break down the meaning of the final answer from the number 45/100 to the meaning 45 times out of 100. *What kind of question could you pose to the class to get everyone thinking about what the 45/100 represents?*

**Student E**



Yesterday the cat door was used 100 times by her cats.

Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.

How many times did Suzi use the cat door? $\frac{45}{100} = \frac{9}{20}$

Explain how you figured it out.

At first I changed the denominators and numerators on $\frac{1}{4}$ and $\frac{3}{10}$ so the were $\frac{25}{100}$ and $\frac{30}{100}$. I added them up and got $\frac{55}{100}$. Then I subtracted 55 from 100, and my answer was 45, and I turned 45 into the fraction that Suzi used the cat door.

$\frac{1}{4} = \frac{25}{100}$ $\frac{3}{10} = \frac{30}{100}$ $\frac{25}{100} + \frac{30}{100} = \frac{55}{100}$ $100 - 55 = 45$

(1) (1) 7 7

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## Implications for Instruction

Students need practice working with fractions in context. They should have a variety of strategies for combining fractions: models, common denominators, changing fractions to decimals or percents. Students should also be able to compare fractions to find out which is larger and subtract fractions from 1 whole.

Some students are still having difficulty choosing operations. Work with bar models might help them to clarify the action of the story problems.

## Ideas for Action Research

## Looking at student work:

Often when planning remediation or helping students who are behind, teachers think about the students who are almost there. What are the few steps they need to be successful? But what is it that the students who are at the lowest end of the spectrum need? How are their issues different?

Sit down with colleagues and examine the following pieces of student work. Consider the following questions:

1. What are the strengths, if any, that the student has? What does the student understand about the meaning of fractions? What does the student know about procedures with fractions? What are the concepts the students understand about the situation? How might these strengths be used to help build their understanding of the whole situation?
2. Is the student making appropriate choices of operations? Do you think the student could pick the correct operation if given a similar problem with whole numbers? What is your evidence? How do students learn to identify the action of the story?
3. How did students use representations? Were the representations accurate? Why or why not? What would have helped the student to improve their representation? Could their representation be modified in some way to solve the problem?
4. What is the role of labels in understanding what is known and what needs to be found? How does using labels help students to understand what they have calculated and interpret the meaning of the calculation? (This seems especially critical in part 4)
5. What misunderstandings does the student have? What skills is the student missing? What does this suggest about a specific course of action to help this student?
6. How are the needs of each of these students the same or different?

After your have carefully looked at each piece of student work, see if you can devise a plan of experiences/ discussions/ tools that might help these students to make more sense of these situations. While you don't have these exact students in your class, each member of the group will probably have students with similar misunderstandings. Identify students who you think are low and plan different approaches for attacking the problems outlined here. Have each person in the group try out a different course of action and report back on the how the lesson or series of lessons effected the targeted students. See if you can all use some similar starting problems and bring work of the students to share. What types of activities or experiences made the most noticeable improvement in student work?

**Arnold**

1. Cindy feeds them on Cat Crunchies.

   Each day Sammy eats $\frac{1}{2}$ of the box, Tommy eats $\frac{1}{8}$ of the box and Suzi eats $\frac{1}{4}$ of the box.

   What fraction of a whole box do the cats eat, in all, each day? $\frac{5}{6}$ x·0 0

   Show how you figured this out.

   $\frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$  $\frac{8}{16} + \frac{2}{16} = \frac{10}{16}$  $\frac{1}{4} \times \frac{10}{10} = \frac{10}{64}$ x·0 0

   $\frac{1}{8} \times \frac{2}{2} = \frac{2}{16}$  $\frac{10}{16} \times \frac{4}{4} = \frac{40}{64}$  $\frac{40}{64} + \frac{10}{64} = \frac{50}{60} \div 2 = \frac{25}{30}$

   $\frac{25}{30} \div 5 = \frac{5}{6}$

2. Tommy and Suzi spend much of each day sleeping.

   Tommy sleeps for $\frac{3}{5}$ of the day and Suzi sleeps for $\frac{7}{10}$ of the day.

   Which of the two cats sleeps for longer? Tommy x·0 0

   How much longer does it sleep each day? $\frac{9}{50}$

   Show how you figured this out.

   $\frac{3}{5} \times \frac{10}{10} = \frac{30}{50}$ = Tommy  x·0 0  $\frac{30}{50} - \frac{21}{50} = \frac{9}{50}$

   $\frac{7}{10} \times \frac{3}{5} = \frac{21}{50}$ = Suzi

3. Cindy's cats often share a carton of cat milk.

   Sammy always drinks $\frac{1}{3}$ of the carton, Tommy always drinks $\frac{5}{12}$ of the carton, and Suzi always drinks $\frac{1}{6}$ of the carton.

   What fraction of the carton of cat milk is left over? $\frac{18}{216}$ x·0 0

   Show how you figured it out.

   $\frac{216}{216} - \frac{198}{216} = \frac{18}{216}$  x·0 0

   $\frac{1}{3} \times \frac{12}{12} = \frac{12}{36}$  $\frac{12}{36} + \frac{15}{36} = \frac{27}{36}$  $\frac{27}{36} \times \frac{6}{6} = \frac{162}{216}$

   $\frac{5}{12} \times \frac{3}{3} = \frac{15}{36}$  $\frac{1}{6} \times \frac{36}{36} = \frac{36}{216}$

   $\frac{162}{216} + \frac{36}{216} = \frac{198}{216}$

4. Cindy's cats love to jump in and out of their cat door.

   Yesterday the cat door was used 100 times by her cats.

   Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.

   How many times did Suzi use the cat door? 77 x·0 0

   Explain how you figured it out.

   $\frac{1}{4} \times \frac{10}{10} = \frac{10}{40}$  $\frac{3}{10} \times \frac{4}{4} = \frac{12}{40}$  $\frac{10}{40} + \frac{12}{40} = \frac{22}{40}$  100 − 23 = 77

   Used fraction. x·0 0