Designing and Teaching Story Problems to Build Solid Conceptual Understanding and Strong Math Practices in Kindergarten

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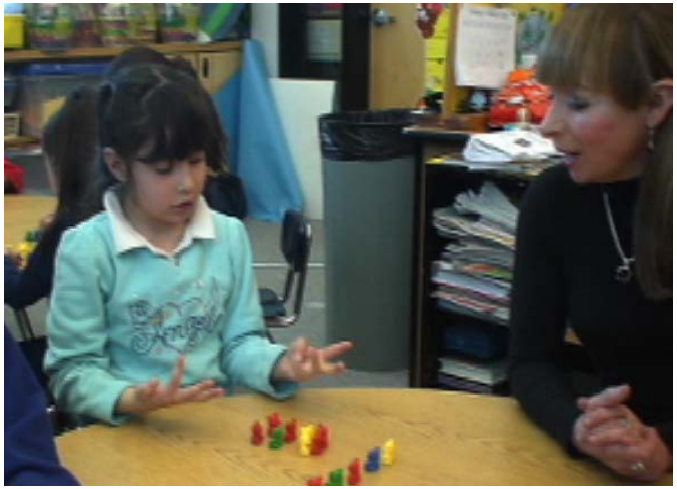

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Imagine children in kindergarten…

- Reasoning about numbers,
- Solving challenging story problems,
- Talking about their math thinking,
- Increasing their vocabulary,
- Learning from their classmates' strategies,
- Solving multiplication, division & fraction problems,
- Making up their own math stories, and
- Feeling confident as math problem solvers!

- These are the goals of story-based problem solving in kindergarten.

Research Background

- 2005-2009: National Science Foundation Center for the Mathematics Education of Latinos/as (CEMELA)
  - Research team goal
    - 1) To improve math teaching in Spanish bilingual primary classrooms through professional development in CGI problem solving, and
    - 2) To understand the mathematical thinking of students during problem solving.
  - Dissertation from the University of New Mexico (2009):
    - EXPLORING THE MATHEMATICAL THINKING OF BILINGUAL PRIMARY-GRADE STUDENTS: CGI PROBLEM SOLVING FROM KINDERGARTEN THROUGH 2ND GRADE

- Publication from kindergarten research:
  - "Fíjense amorcitos, les voy a contar una historia": The power of story to support solving and discussing mathematical problems with Latino/a kindergarten students. Turner et al., 2009, NCTM. (See Bibliography and COSA website.)

Learning Theory in Math Education

- The brain learns by sense-making. (Cognitive Psychology)
  - How children learn led to the first National Council of Teachers of Mathematics Standards in 1989.
- Children make sense of new concepts by incorporating them into what they already know about the world. (Sociocultural Theory)
  - New knowledge is built on old and communicated by language.
- Children co-construct new knowledge through social interactions. (Social Constructivism)
  - As they explain their thinking, they consolidate and organize their own thoughts.
  - As they listen to others, they modify what they know.
- ***Children will "learn math with understanding" if they are supported from below rather than dragged from above***.

Key Ideas About Teaching Through Stories

- Stories bridge informal math experiences with formal school learning.
- Stories from children's own cultural and linguistic backgrounds provide an ***Opportunity to Learn***.
- Cognitively Guided Instruction (CGI) is a framework for developing a range of story types.
- Story problems support literacy development.
- Making sense of stories leads to understanding formal representations.
  - $3 + 4 = \square + 5$ $\square + 2 = 6$
  - These do not make sense to children without a context.

Workshop Objectives

- Participants will:

- 1. Learn about the Cognitively Guided Instruction* (CGI) framework.
  - CGI problems are found in state-wide curricular materials.
  - They are also found in the glossary of the Common Core State Standards for Math.

- 2. Practice writing different types of story problems.

- 3. Explore teaching strategies for CGI storytelling.

*Carpenter, Fennema, Franke, Levi & Empson (2015, 1999)

Three Research Articles

- “A Problem is Something You Don’t Want to Have”: Problem Solving by Kindergartners. Outhred & Sardelich, 2005.

- “Fijense amorcitos, les voy a contar una historia”: The Power of Story to Support Solving and Discussing Mathematical Problems among Latino and Latina Kindergarten Students. Turner, Celedon-Pattichis, Marshall & Tennison, 2009.

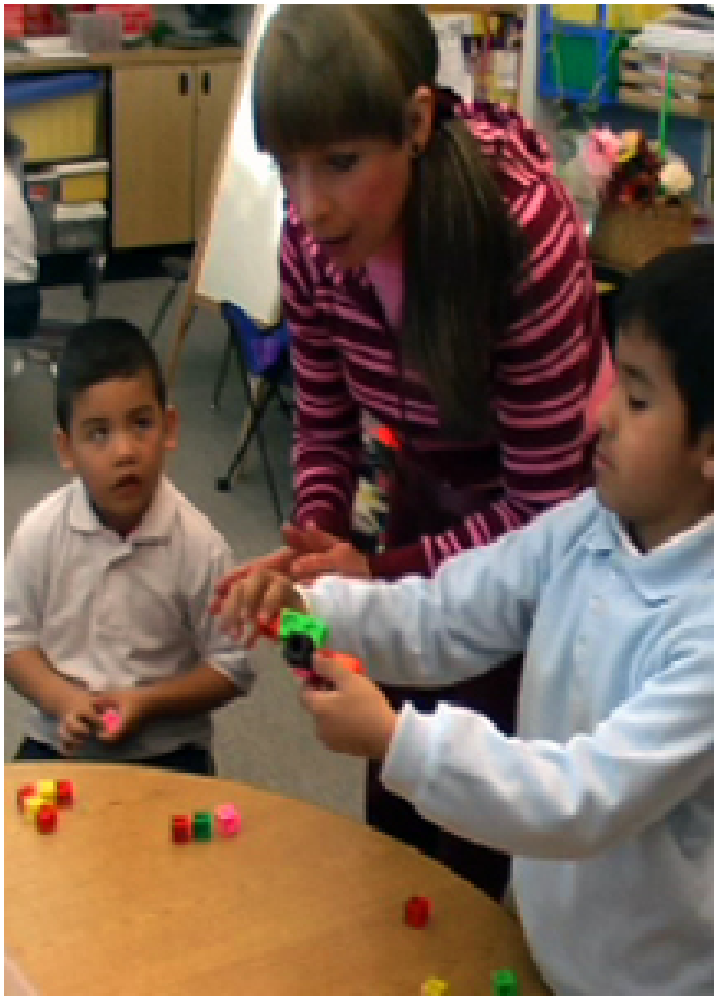
- Mathematical Graphic Organizers. Zollman, 2009.

Australian Study of Kindergarten Problem Solving (Outhred & Sardelich, 2005)

- Kindergartners successfully solved a range of challenging word problems.
- Stories built on prior knowledge and used the language children were used to hearing.
- Students drew pictures to find solutions.
- Children talked about their thinking.
- Children posed their own problems.
- Teachers built understanding in whole class discussions.
- Writing number sentences was developed from children's strategies.

University of New Mexico Research in "Fíjense amorcitos…" (Turner et al., 2009)

- Students developed and used their *own strategies* to solve problems.
- They made sense and reasoned quantitatively to find solutions.
- Their own strategies became the basis for writing with math symbols.
- Low SES immigrant students were highly successful.



“Mathematical Graphic Organizers”
(Zollman, 2009)

- **Drawing is a powerful strategy in problem solving.**
  - Part of Common Core Math Practice Standards, “#4 Modeling”
  - Helps children see number relationships
  - Helps children explain their thinking
- A graphic organizer with drawing guides older children through the problem solving process.

What is Cognitively Guided Instruction (CGI)?

- Developed by Carpenter, Fennema, Franke, Levi and Empson at the University of Wisconsin
- First published in 1999, 2nd edition 2015
- Developed from work in math education research
- Framework for understanding young children's mathematical thinking
  - Uses teaching methods that respond to that thinking
- Focuses on story types and student strategies
  - Direct modeling, counting, developed strategies

- Used for Marshall dissertation analysis

CGI Storytelling and the Common Core State Standards in Math Practice

#1. Make sense of problems and persevere in solving them

- CGI emphasizes sense making as the critical first step.

#2. Reason abstractly and quantitatively

- Children develop their own strategies to find solutions.

#3. Construct viable arguments and critique the reasoning of others

- Children listen to each other, discuss solutions and share strategies.

#4. Model with mathematics

- Children find answers by modeling and drawing.

Additionally, math practices in kindergarten…

- Encourage higher order thinking
  - For example: Analyzing and evaluating (Bloom's)
- Develop language and vocabulary
- Emphasize good listening skills
- Support social skills, self regulation, and executive function

Two Basic Categories of CGI Problems: Action and Relationship

JOIN

Connie had 8 marbles. JUAN gave her 5 **more marbles**.

How many marbles does Connie have now?

ACTION (easier to model)

COMPARE

Juan has 5 marbles. Connie has 8 more marbles than Juan.

How many marbles does Connie have?

RELATIONSHIP (harder to model)

CGI Problem Types with Increasing Complexity: Part 1

(left to right, top to bottom)

Join	*(Result Unknown)* Connie had 5 marbles. Juan gave her 8 more. How many marbles does Connie have altogether?	*(Change unknown)* Connie has 5 marbles. How many more marbles does she need to have 13 altogether?	*(Start Unknown)* Connie had some marbles. Juan gave her 5 more. Now she has 13. How many marbles did Connie have to start?
Separate	*(Result Unknown)* Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?	*(Change unknown)* Connie had 13 marbles. She gave some to Juan. Now she has 8 marbles left. How many marbles did Connie give to Juan?	(*Start Unknown)* Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start?
Part-Part Whole	*(Whole Unknown)* Connie has 5 red marbles and 8 blue marbles. How many marbles does she have altogether?	*(Part Unknown)* Connie has 13 marbles. 8 are blue. The rest are red. How many red marbles does Connie have?	
Compare	*(Difference Unknown)* Connie has 13 marbles. Juan has 8 marbles. How many more does Connie have than Juan?	*(Compare Quantity Unknown)* Juan has 8 marbles. Connie has 5 more than Juan. How many marbles does she have?	*(Referent Unknown)* Connie has 13 marbles. She has 5 more than Juan. How many marbles does Juan have?

CGI Multiplication and Division Problem Types: Part 2

Multiplication	Megan has 5 bags of cookies. There are 3 cookies in each bag. How many cookies does Megan have in all?
Measurement Division	Megan has 15 cookies. She puts 3 cookies in each bag. How many bags can she fill?
Partitive Division	Megan has 15 cookies and 5 bags. She wants to put the same number of cookies in each bag. How many cookies should she put in each bag?

Why Different Problem Types?

- Story structure influences student comprehension.
  - Making sense depends on the number relationships outlined in the structure.
  - Children think about the CGI types in different ways because they have different structures.
- Starting with math operations ignores the structure.
- Using all types develops mental flexibility.
- Mental flexibility underlies algebraic thinking.

Different CGI Problem Structures for Young Children

Different CGI problem structures for young children:

- Action problems:
  - Join (3), Separate (3),
  - Multiply (1), Divide (2)
- Relationship problems:
  - Part-Part-Whole (2),
  - Compare (3)

Handout 1: The fourteen problem types.

Handout 2: A blank form for writing problems.

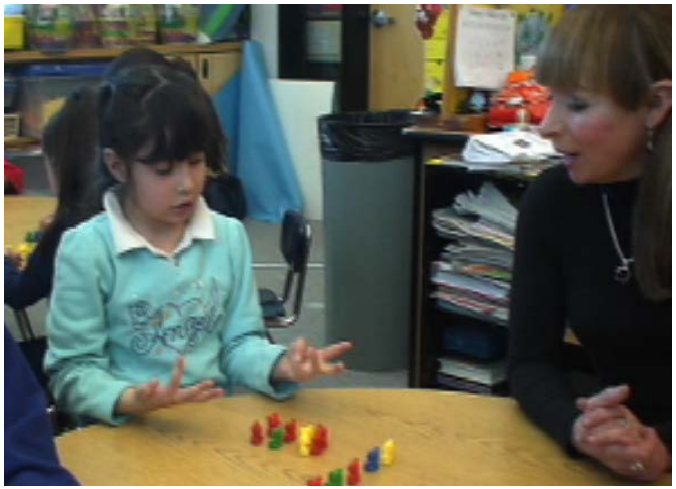
Activity: Modeling Solutions to Story Problems (20minutes)

1. Try to solve all the Cognitively Guided Instruction problems on the handout the way you think your students would solve them.

2. Use cubes, counters or pencil/paper.

3. Share and discuss with your neighbors.

4. Would anyone like to stand and share?



How to Teach with CGI Problems: Ms. Byron Begins a Math Lesson

- Online CGI video from “Children’s Mathematics.”

- Note:
  - Problem is posed in a conversational style.
  - It connects to children’s experience with literacy.

(Video clips available through purchase of text from Heinemann.)

Ms. Byron's Problem Posing

- She unpacked the problem.
  - "The goal is to provide opportunity for students to make sense of the problem context, not to walk them through how to solve the problem" (Carpenter et al., 2015).
  - She did not focus on "key words",
    - Or the question,
    - Or the the operations students should use.
    - Or discuss strategies to lead them toward the solution.

Instead, she…

- Focused on story comprehension,
- Concentrated on the connection between the story and the math,
- Explored the numeric relationships,
- Supported each student's participation, and
- Had children talk about the story and add details.
  - Then she directed them to work with each other to develop strategies and find solutions.

After Students Work Together: Elicit Student Thinking

- A major goal of CGI is to help teachers understand students' math thinking.
- During and after problem solving, consistently ask students to
  - talk about their strategies,
  - explain what they have done, and
  - say why they think their answers are correct.
- Listen and observe what students are doing.
- Build group understanding from students' own words and actions.

Summary of a CGI Lesson

- Create a conversational, storytelling atmosphere; engage the students.
- Let them add details.
- Have students draw or model.
- Guide and scaffold rather than direct.
- Believe in children's sense-making ability.
- Build on their strategies and explanations.
- Make listening to each other a major goal.
- ***Let students decide if the answers are correct or not***.

Strategies to Support Executive Function

- Ms. Bryon could use Think-Pair-Share.
- She could also give them a little more wait time.
- Students could solve one of their own problems first.
- Calling on students could be with Stick Picks.
- Students could solve a problem as a group on the floor with white boards.
- Ms. Bryon could do whole group answers as a formative assessment.

Sense-Making Comes First in Problem Solving

- Math teaching and learning begin with sense making.
- Only when students have
  - made sense of the problem,
  - found a strategy to solve it with drawing, manipulatives, or role play,
  - have explained their thinking,
  - have listened to other ways of solving the problem from their peers, and
  - decided on the correct answer,
- Can they make a strong connection between the math concept and a math equation.

Kindergarten Students and Math Equations

- Kindergarten students do not need to know how to write the math equations for all the stories they can solve.
- Even though they can solve simple multiplication and division problems,
- they will learn to write these equations later.
  - If they ask (and they will) show them how to write the number sentences.
- Your goal is to build the foundation for future learning.

Importance of Partitive Division Problems Beginning in Kindergarten

- They lay the foundation for understanding fractions.
- They emphasize sharing equally.
- Learning to share is a major social goal of kindergarten.
- They build on children's out-of-school experiences.
- Children arrive in kindergarten with the concept of "half" firmly in place.

Examples of Student Drawings for Partitive Division Problems

- Kindergarten:
  - 4 cookies on a plate that 2 children share
  - Then 3 cookies on a plate that 2 children share
- Problem for 1st and 2nd:
  - 6 cookies on a plate that 4 children share
- Problem 3rd:
  - 3 dozen cookies that 21 students plus the teacher share
- (Show .pdf file)

Wrapping Up

- Math in kindergarten is more than learning to count.
- ***It lays the foundation for all future math learning.***
  - Stories bridge children's lived experiences to number concepts.
  - Stories support both math and literacy.
  - Drawing/modeling is a critical first step in sense making.
  - Number sentences should reflect children's own strategies.
- Solving story problems, and telling their own stories,

engage, empower and motivate children.

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