Mathematics for Every Student

Responding to Diversity, Grades Pre-K–5

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"Fíjense amorcitos, les voy a contar una historia": The Power of Story to Support Solving and Discussing Mathematical Problems among Latino and Latina Kindergarten Students

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A CENTRAL tenet of *Principles and Standards for School Mathematics* (NCTM 2000) is that students at all levels, even young children, should participate in solving problems and explaining and justifying their mathematical thinking (NCTM [2000]; see also Cobb, Wood, and Yackel [1993]; Lampert [2001]). Previous research has demonstrated that children as young as kindergarten (Carpenter et al. 1993; Outhred and Sardelich 2005) and first grade (Secada 1991; Villaseñor and Kepner 1993) can solve a broad range of mathematical problems, often by modeling the quantities and relationships involved (see also Carpenter et al. [1999]). For example, a five-year-old might solve a simple multiplication problem, such as "Sara had three pockets. She put two pennies in each pocket. How many pennies does she have in all?" by drawing three pockets, placing two counters (pennies) in each of the pockets, and then counting all the counters to determine that she has six pennies altogether. Carpenter and his colleagues (1993) found that kindergarteners who had repeated opportunities to solve a variety of basic word problems demonstrated remarkable success on an end-of-the-year assessment. Almost half of the seventy students interviewed used valid strategies on all the word problems, which included multiplication, division, and multistep problems; and most students were successful on very basic problem types (e.g., subtraction).

Although we know that young children are capable of solving problems and engaging in mathematical reasoning (Tang and Ginsburg 1999), we know less about how early primary-grade teachers support problem solving and mathematical discussion, particularly in culturally and linguistically diverse classrooms, as well as in classrooms in which students have a wide range of prior mathematical experiences. Teachers may worry

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that until young children have mastered certain basic skills (e.g., counting, recognizing numbers, and comparing sets), they may struggle to solve problems and explain their reasoning. Another concern is that if students are learning the language of instruction, as with many children who are English language learners (ELLs), such language-intensive activities as interpreting, solving, and discussing word problems in one's second language may be too challenging (Iddings 2005).

Overview of Classroom Study

The goals of this study were to document the development of problem solving among Latino and Latina kindergarten students and to identify specific instructional practices that teachers used to help students solve problems and communicate their mathematical thinking.

The goals of this study were to document the development of problem solving among Latino and Latina kindergarten students and to identify specific instructional practices that teachers used to help students solve problems and communicate their mathematical thinking. Specifically, this article describes practices that draw on students' cultural and linguistic knowledge and experiences (e.g., González, Moll, and Amanti 2005). We focused on two kindergarten classrooms, both in schools with predominantly Latino and Latina student populations (87 percent and 72 percent), in which more than 90 percent of the students qualified for free or reduced-price lunch. In Ms. Arenas's[1] classroom, all students were native Spanish speakers with varying degrees of English language proficiency. Ms. Arenas, also a native Spanish speaker, followed a bilingual model of instruction. Each morning, students worked in their native language on reading, writing, and mathematics activities. During the afternoon, students participated in integrated projects and additional literacy-related tasks in English. Ms. Field, who had training in English as a second language (ESL) strategies, taught mathematics in English. Half of Ms. Field's students were ELLs, and the rest were native English speakers.

We selected these two classrooms because both teachers had participated in Cognitively Guided Instruction (CGI), a summer professional development workshop that focused on the development of young children's mathematical thinking about basic operations (see Carpenter et al. [1999]), and they were interested in implementing problem-solving lessons with their students. We visited each classroom biweekly to videotape lessons. At the beginning of the year, we selected sixteen students (eight from each class) who represented a range of achievement levels to participate in a problem-solving interview assessment (Ginsburg et al. 1983). This preassessment included both counting and problem-solving tasks (e.g., join, separate, multiplication, and division word problems). The researchers presented all problems orally, and students had access to multiple problem-solving tools (counters, cubes, paper, and pencils). At the end of the year, we administered a similar postassessment that included a broader range of problems. We asked students to explain their reasoning for each problem.

In the sections that follow, we present a brief overview of the preassessment results and then describe typical problem-solving mathematics lessons in each of the classrooms. We then present detailed classroom examples to illustrate specific instructional strategies that teachers used

1. All names are pseudonyms.

to help all students make sense of and solve problems and explain their mathematical reasoning. We conclude with a discussion of students' performance on the postassessment to demonstrate the impact of these instructional strategies on students' learning.

Beginning-of-the-Year Problem-Solving Assessments

As is typical of many kindergarten classrooms, the children began the school year with a range of mathematical experiences and proficiencies. Most of the students could count a small set of objects (three to eight items) and recognize some numerals from 1 to 10, as measured by a Kindergarten Developmental Progress Record that the district administered. However, at the beginning of the year, several students could not count past two or three and did not demonstrate one-to-one correspondence, even with numbers under five. As Ms. Arenas noted, "My students, some of them didn't recognize any of the numbers, higher than three they probably didn't know. Some of them counted, one, two, [and then] one hundred! They didn't know how to count, so I had to develop that little by little at the same time that I was doing [problem solving]." In problem solving, slightly less than half ($n = 7$) of the sixteen students who participated in the preassessment successfully solved a basic addition problem (six jelly beans plus three more), and slightly more than half ($n = 10$) solved a basic subtraction problem (five pennies take away two).

Other problems on the preassessment—such as the multiplication, division, and comparison problems—were significantly more difficult; only two or three of the sixteen children used valid solution strategies for these problems. As is typical of young children, students provided short and often vague descriptions of their thinking, using such phrases as "I just knew it" and "I counted" to justify their answers. We include this overview to establish that students reflected a typical range of understanding, and if anything, that some students began the year with less-developed counting skills than kindergarten teachers might expect.

Classroom Learning Context

Ms. Arenas and Ms. Field drew on a variety of instructional formats in their problem-solving lessons. Although the researchers did not provide Ms. Arenas and Ms. Field with specific guidelines for instruction, the researchers encouraged the teachers to use information about children's thinking and about basic problem structures (e.g., different types of addition and subtraction problems) to plan and adapt problem-solving tasks. Common to each of their lessons was that the teacher orally presented a word problem to students and then encouraged students to solve the problem in ways that made sense to them. Table 2.1 gives examples of problems that the teachers presented. The students often used such concrete materials as counters and cubes or drew pictures on small whiteboards to support their reasoning (fig. 2.1 and fig. 2.2). After most students had solved a problem, the teachers facilitated group discussion in which several students shared their strategies.

The teacher orally presented a word problem to students and then encouraged students to solve the problem in ways that made sense to them.

Table 2.1
Examples of "Mathematical Stories" That Ms. Arenas and Ms. Field Presented to Their Students

Problem Type		"Mathematical Story"
Join (result unknown)	a.	Ms. Arenas went to the market because she had to buy oranges. She got 4 oranges and put them in her basket, and then her son José got another 4 oranges, and he put those in the basket too. How many oranges did they have in the basket?
Join (change unknown)	b.	Marian goes to the store, and she wants to buy a bag of candies; but the bag of candies costs 8 dollars, and she only has 5 dollars. How many more dollars does she need?
Multiplication	c.	You and your two best friends are playing at recess, and you find some pennies outside on the playground. You each find 2 pennies and you each put the pennies in your pocket. How many pennies did you find altogether?
Separate (result unknown)	d.	Let's say that Alfredo had 9 marbles, and then he gave 4 marbles to Cesar. How many marbles does he have left?
Division (partitive)	e.	Ms. Arenas has 15 little puppies, and she wants to give them to Paolo, Daniel, and Mónica. What can she do so that they each get the same number? How many should she give to each of them?

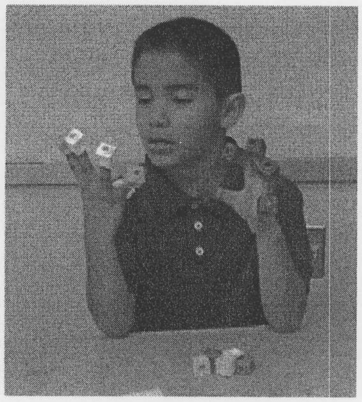

Fig. 2.1. Orlando uses cubes to solve a multiplication problem.

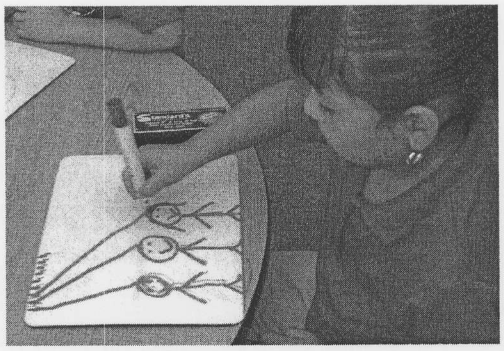

Fig. 2.2. Dora draws a picture to solve a division problem.

In some instances, particularly early in the year, the teachers worked on problem solving with one small group of students at a time while other students participated in center activities. Typically, the teachers grouped students heterogeneously, although sometimes the teacher grouped specific students because she believed that they would benefit from working collaboratively on a particular type of problem. On other occasions, the teachers presented the entire class with a problem, and students worked individually or with a partner to generate a solution. Both teachers also used center activities that involved building and counting sets to support students' number sense and counting skills. The teachers did not wait until students had mastered a set of basic skills to begin problem-solving lessons. Instead, they introduced problem solving at the very beginning of the year and used contextual problems "to strengthen students' counting ... numerical skills and reasoning" (interview with Ms. Arenas).

The teachers did not wait until students had mastered a set of basic skills to begin problem-solving lessons. Instead, they introduced problem solving at the very beginning of the year and used contextual problems "to strengthen students' counting ... numerical skills and reasoning."

Powerful Instructional Strategy: The Use of Story

As the researchers analyzed lessons from each classroom, they identified a number of instructional strategies that the teachers used to support students' learning. Although a discussion of all strategies is beyond the scope of this article, the focus here is on a particular feature of the lessons that was especially generative: the use of story as a way to organize problem-solving sessions and scaffold kindergarten students' understanding while they solved and discussed basic word problems. Using children's stories to generate mathematical problems and to connect students' cultural knowledge and experiences with their mathematical activity has been documented in prior research (Lo Cicero, Fuson, and Allexsaht-Snider 1999; Lo Cicero, De La Cruz, and Fuson 1999). Our work extends previous research by contributing a detailed analysis of how the use of stories supports the learning of Latino and Latina kindergarten students, in particular, how it supports students while they learn to make sense of, represent, and explain their thinking about mathematical problems.

Using children's stories to generate mathematical problems and to connect students' cultural knowledge and experiences with their mathematical activity has been documented in prior research (Lo Cicero, Fuson, and Allexsaht-Snider 1999; Lo Cicero, De La Cruz, and Fuson 1999).

Using Stories to Generate Mathematical Problems

The teachers in this study used stories in different ways. Ms. Arenas typically began mathematics lessons by gathering students on the carpet and telling them to listen carefully because she was about to share a story with them. Her stories drew on students' shared experiences (e.g., field trips and class parties), events and activities in the community (e.g., going to the fair and purchasing fruit at the local market; see table 2.1, problems a and b), and her own experiences outside school (e.g., going to the park with her nephew). For example, during a lesson early in the year, she began with a story about a woman who lived directly behind the school and her cats.

Ms. Arenas: Fíjense, mis niños, fíjense que la señora, aquí atrás de la escuela, que vive aquí atrâs. ...

(Listen, my dear children, you know the woman, the woman here behind the school, that lives right here in the back [of the school].)

Students: ¡Sí! *(nodding heads, indicating that they know the woman to whom she is referring)*

Ms. Arenas: La señora tenía tres gatos. *(The woman had three cats.)*

Students: ¿¡Tres gatos!? *(Three cats?)*

Ms. Arenas: Sí, pero luego, su hija le regaló otros tres. Y luego, todos los gatos vienen a jugar aquí a la escuela. ¿Cuántos vendrán? ¿Cuántos gatos?

(Yes, but then, her daughter gave her another three. And then, all the cats came here to play at the school. How many came? How many cats?)

Student: Seis!

Ms. Arenas: Escuchen, la señora tenía tres gatos, y luego su hija le regaló otros tres, ¿cuántos tendrá ahora? Piensen, y ahora me van a decir.

(Listen, the woman had three cats, and then her daughter gave her another three; how many would she have? Think about it, and then you are going to tell me.)

Later in the same lesson, Ms. Arenas presented another story about going to the pet store and purchasing six puppies and then giving several of the puppies away. She included rich contextual information, including such details as what the puppies looked like or how her nephew played with the puppies at the park. Another distinctive feature of the stories that Ms. Arenas told was that she presented them in a very informal, conversational manner. That is, Ms. Arenas framed her talk as telling a story and not as presenting a mathematics problem. Students responded accordingly, making comments, adding their own details, and asking questions.

Ms. Field also told stories about students and class activities to frame mathematical problems. She used such events as children's finding pennies on the playground and a game of marbles between two students to structure multiplication and subtraction problems, respectively (table 2.1, problems c and d). In addition, Ms. Field used conversations about books as contexts for generating mathematical problems. For example, during a lesson in the spring, Ms. Field shared a book about honeybees with her students. She read small portions of the book, shared pictures of beehives, and even passed around a piece of honeycomb for students to examine. Ms. Field used the book to elicit a conversation about honeybees; and throughout the conversation, students shared their personal experiences and existing knowledge.

At one point, Bernardo noted that the bees' honeycomb was the same shape as the hexagon pattern block that students often used to create geometric designs (see fig. 2.3). This realization sparked a discussion of how many sides a hexagon has and ultimately, the multiplication problem "Then how many sides would three hexagons have?" Although Ms. Field may have anticipated this mathematical connection, the multiplication problem arose spontaneously as students talked, told stories, and asked questions about honeybees. Of significance is that in addition to using students' home and school experiences as contexts for mathematical problems, Ms. Field used a story and an informal conversation about honeybees to create a new, shared experience (i.e., examining the geometry of a honeycomb) that then generated a series of problems for students to solve.

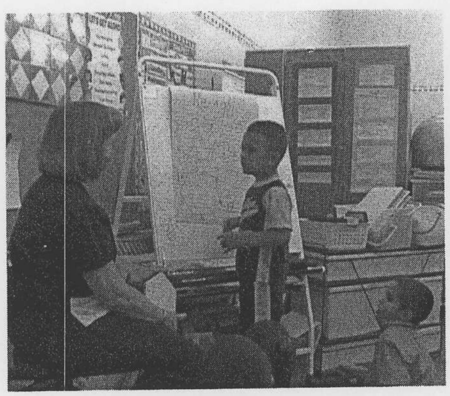

Fig. 2.3. Bernardo makes a connection between a honeycomb and hexagons.

The previous examples provide a sense of how Ms. Arenas and Ms. Field used stories to generate mathematical problems in their kindergarten classrooms. The following series of classroom episodes describe how this instructional practice supported students while they made sense of, represented, solved, and communicated their reasoning about a variety of problems.

Episode 1. Teachers Use Stories to Draw on Students' Funds of Knowledge—Familiar Ways of Talking, Cultural Experiences, and Activities

Ms. Arenas and Ms. Field shared mathematical stories with their students in an informal, conversational manner. At times, the teachers and the children coconstructed the stories while students made such decisions as how many children would share a box of twelve cookies or how much money a toy airplane should cost. For example, during a lesson in November, Ms. Arenas and her students constructed the following story about shopping at the local produce market.

Ms. Arenas:	El otro día, fui al mercado con mi hijo Eduardo, ¿Y qué creen que compramos? *(The other day, I went to the market with my son Eduardo, and what do you think we bought?)*
Student:	¡Naranjas! ¡Naranjas? *(Oranges! Oranges!)*
Ms. Arenas:	Muy bien. Yo agarré una bolsa y metí seis naranjas en la bolsa. Y luego, Eduardo fue por más naranjas. ¿Cuántas creen que él agarró? *(OK, I got a bag and put six oranges in the bag. And then, Eduardo went to get more oranges. How many do you think he got?)*
Students:	Seis! Cuatro! Metió seis más! *(Six! Four! He put in six more!)*
Ms. Arenas:	Muy bien, metió seis más. Asi que yo puse seis naranjas en la bolsa y luego Eduardo metió seis más. ¿Cuántas naranjas están en la bolsa? *(OK, he put in six more. So I put six oranges in the bag, and Eduardo put in six more. How many oranges are in the bag?)*

The fact that the teachers and students generated mathematical problems from authentic conversations is important, because the teachers drew on ways of talking and negotiating meaning that were familiar to children.

The fact that the teachers and students generated mathematical problems from authentic conversations is important, because the teachers drew on ways of talking and negotiating meaning that were familiar to children. That is, all children have experience telling and listening to stories and using stories to communicate meaning, a practice that is particularly prevalent in Latino families (Delgado-Gaitan 1987; Villenas and Moreno 2001). Teachers also drew on students' cultural knowledge and experiences by creating stories that reflected familiar situations. As Ms. Arenas noted,

> [Children] bring rich experiences in going to the market with their parents. ... The market experience, the open market

> experience, [experiences] with money and how much do you pay for this or that. ... They have other cultural experiences, too. A lot of times they plant with their parents, and even counting the seeds or transferring the seeds is mathematics.

When teachers draw on language and ways of talking that are familiar to children and use relevant contexts to introduce new ideas (for example, using a story about planting seeds to frame a multiplication problem), this teaching practice supports students' understanding (Dalton 1998).

Episode 2. Teachers Use Stories to Help Students Represent Mathematical Relationships and Connect Multiple Representations

Teachers also used stories to support students while they learned to represent quantities and mathematical relationships in different ways (e.g., with objects, drawings, and symbols). The following episode from Ms. Field's classroom illustrates how the teacher continuously referred to the story to clarify the meaning of different representations.

Ms. Field: OK, let's try a different one. You ready? OK, let's just say that Alberto had nine marbles, and he's playing marbles, and he gave four to Cesar. How many marbles does Alberto have left?

Students began to work on the problem, and Ms. Field talked with them about their strategies. Alberto held up nine fingers, and then lowered four, one at a time, to figure out how many marbles he had left. Iván drew nine circles on his whiteboard, crossed out four, and counted those that remained. After most students had solved the problem, Ms. Field asked Verónica to share her strategy.

Ms. Field: Verónica?

Verónica: Take four and it would be five.

Ms. Field: Ah, draw that again, and show us. Let's look at Verónica's *(positions Verónica's whiteboard so that other students can see her work)*. So what did you draw first?

Verónica: Nine *(points to nine circles that she has drawn)*.

Ms. Field: And then what did you do?

Verónica: Then I erased these ones *(points to four circles, and begins to erase them)*, and it would be five.

Ms. Field: Go ahead, go ahead.

(Verónica erases four of the nine circles that she has drawn on her whiteboard.)

Ms. Field: *(Ms. Field points to the circles that Verónica has erased.)* So those are the ones that he gave to Cesar?

(Verónica nods.)

Ms. Field: And you have how many left?

Verónica: Five.

Ms. Field: And guess what, that's a takeaway, that's a minus *(begins to write a number sentence on the whiteboard).* It's nine, he started with nine marbles *(writes the number 9, and then above the number draws nine circles).* Then give away four marbles (she erases four of the circles), so I take away four (*writes "– 4" next to the* 9), that equals five. That's the number sentence! (*She completes the number sentence by writing* "= 5.")

In this example, we see how Ms. Field referred to the story both to clarify Verónica's pictorial representation (i.e., "So those [the circles she erased] are the ones that he gave to Cesar?") and to introduce a new representation, a number sentence, that students could use to model the problem. As she wrote the number sentence on the board, Ms. Field linked each element to a part of the story. For example, as she wrote "– 4" she reminded the students that Alberto gave away four marbles, which she also represented by erasing four of the nine circles that she had drawn (see fig. 2.4).

Fig. 2.4. Verónica shares her solution with the class.

The teachers used stories to help students make sense of different representations—those used by other students and those that the teacher introduced—and to connect multiple representations for a particular problem. Consider the following example from a lesson in Ms. Arenas's classroom.

The teachers used stories to help students make sense of different representations—those used by other students and those that the teacher introduced—and to connect multiple representations for a particular problem.

Ms. Arenas: OK, mis niños, ahora viene otra historia, fíjense. Oscar fue al parque, y vio cuatro gatos. Escuchen. Oscar fue al parque y vio cuatro gatos; ¿cuántas patas vio? Vio cuatro gatos! ¿Cuántas patas vio? Piensen.

(OK, dear children, here comes another story. Listen, Oscar went to the park, and he saw four cats. Listen. Oscar went to the park and he saw four cats; how many legs did he see? He saw four cats! How many legs did he see? Think about it.)

After students worked on the problem, Ms. Arenas asked several students to share their solutions with the group. Daniel came to the whiteboard first and drew four sets of four tally marks to show how he figured out that the four cats had sixteen legs (fig. 2.5). Ms. Arenas asked him about his drawing, and he explained that each group of four tally marks represented one cat. He then counted all the legs and said that there were sixteen legs altogether. Next Ms. Arenas invited other comments about the problem.

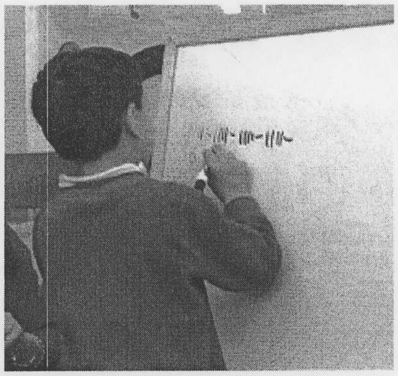

Fig. 2.5. Daniel uses tally marks to represent four cats with four legs each.

Ms. Arenas: Muy bien. ¿Hay alguien que quisiera hacer algo más con este problema? ¿A ver Emilio?

(Very good. Is there someone who would like to do something else with this problem? Let's see, Emilio?)

Emilio: ¡En números!

(With numbers!)

Ms. Arenas: ¿Cómo lo vas a poner Emilio?

(How are you going to put it, Emilio?)

Student: Cuatro más cuatro más cuatro más cuatro!

(Four plus four plus four plus four!)

(Emilio goes up to the white board and writes 4 + 4 + ...)

Ms. Arenas: Muy bien. Ahora, ¿cuántas patas hay ahí?

(Very good. Now, how many legs are here?) *(points to the next group of four tally marks)*

(Emilio continues writing + 4 + 4.)

Ms. Arenas: Muy bien, ¿y ahora qué pones? ¿Es igual a cuántos? ¿Cuántos te habían dado?

(Very good, and now what are you going to put, it's equal to how many? How many did you get?)

(Emilio writes = 16.)

Ms. Arenas: Mira como lo hizo Emilio, él puso cuatro patas de un gato *(points to four tallies, then to the number 4)*, más *(points to the plus sign)* cuatro patas del otro, más cuatro patas del otro, más cuatro patas del otro, son dieciséis *(continues to point back and forth between each group of four tallies and each number 4 in the number sentence)*. Muy bien, esto es lo que se llama una suma larga.

(Look at how Emilio did it, he put four legs from one cat, plus four legs from another, plus four legs from another one, plus four legs from the other one, it's sixteen. Very good, this is what is called a long addition.)

In this episode, we see how Ms. Arenas repeatedly referred to the story context in an effort to help students connect Daniel's tally-mark representation with Emilio's number-sentence representation. She carefully pointed back and forth between a set of four tally marks and a 4 in the number sentence while she explained that each representation stood for the four legs on one cat and that the addition symbol indicated that they should add the legs from all four cats (fig. 2.6). Learning to represent quantities and mathematical relationships and to connect multiple representations of the same situation are important mathematical goals for young children (NCTM 2000). Access to multiple representations may be especially helpful for ELLs because each representation creates a new opportunity for students to make sense of the problem.

Episode 3. Teachers Use the Structure of Stories to Support Students While They Learn to Explain Their Thinking

Another way that the teachers used stories to support mathematical learning was by drawing on the structure of stories to scaffold students'

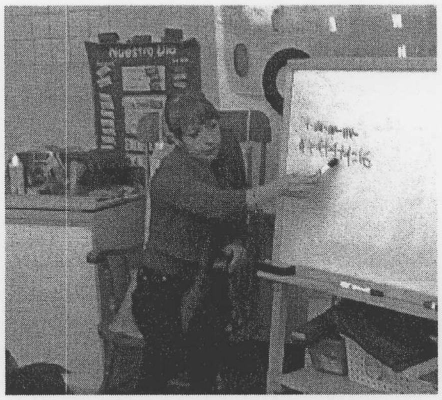

Fig. 2.6. Ms. Arenas uses the story to help students connect multiple representations.

language while students learned to explain their thinking. When students struggled to explain their ideas, the teachers often reminded them of the story context and then used the story as a framework to guide the students while they explained the steps that they took to solve the problem. For example, Ms. Field presented her class with the following story about a game of marbles between Sarita and Kyle:

> Sarita had eight marbles, and then Sarita gave some of those marbles to Kyle. She gave them away. And now she only has four left. So how many did she give to Kyle? Go ahead, try it.

She repeated the story several times, clarifying the quantities involved, and then allowed students time to work. After several minutes, Penny asked whether she could share her solution with the class.

Ms. Field: Sure, come on up Penny. Everybody, eyes up here. Tell us what you did.

Penny: First I started with four, and then I started with four more, then I counted then it made nine, and then I counted, and then ... *(she pauses, seems uncertain, and looks up at Ms. Field).*

Ms. Field: OK, Penny, wait a minute, let me tell you the problem one more time. We said that Sarita had eight marbles, and then she gave Kyle some, and she had four left. So how many did she give him?

Penny: Four, four. *(Penny points to her picture. She has drawn a line of eight marbles, with four marbles on one side of her whiteboard and four marbles on the other.)*

Ms. Field:	So she gave him four marbles?
Penny:	Yes.
Ms. Field:	And then how many did she have?
Penny:	She gave [him] four and had four more left. Cause [it's] like four and four. Eight *(points to the two groups of four on her board).*

Ms. Field then restated Penny's solution, emphasizing that Sarita started with eight marbles and gave away four, and she then had four left and that Penny figured out that she gave away four because she knew that four and four is equal to eight. In this episode, Ms. Field drew Penny back into the story context ("Wait a minute.... We said that Sarita had eight marbles ...") when Penny seemed unsure how to explain her strategy. Ms. Field then asked focused questions about various parts of the story ("So how many did she give him?") as a way of guiding Penny to draw on the story to help her explain her thinking. In the end, when Ms. Field restated Penny's ideas, she again offered Penny a model of how she might use the story to frame her explanation.

When we compared lessons from various points during the year, we noticed that although the teachers initially provided substantial guidance and used modeling to help students communicate their reasoning, many students began to contribute clearer and more complete explanations as the year progressed. Almost without exception, students grounded their explanations in the mathematical stories.

We suspect that repeated teacher-student interactions in which the teacher used the structure and events of stories to guide students' explanations, along with repeated opportunities to hear explanations from their peers, helped students learn to explain their thinking. For example, in the following episode from a lesson in the spring, two girls in Ms. Arenas's class presented their solutions to a multistep word problem: "Ms. Arenas had two boxes of little chocolate eggs. In each box, there were four eggs. Then Ms. Arenas ate two of the eggs. How many were left?" The first student to explain, Delia, was initially hesitant; and Ms. Arenas drew on the structure and the details of the story to help her begin.

Delia:	Primero Ms. Arenas se comió ... *(First Ms. Arenas ate ...)* *(She pauses, not sure what to say next.)*
Ms. Arenas:	Bueno. Primero, ¿cuántos tenía en cada cajita? *(OK, first how many did I have in each box?)*
Delia:	Cuatro. *(Four.)*
Ms. Arenas:	OK, muy bien, y pusiste cuatro en cada cajita. Y luego me comí dos ... *(OK, very good, and you put four in each box. And then I ate two ...)*

Delia: Y le quedaron seis.

(And there were six left.)

(Delia points to the picture that she has drawn. It has two groups with four circles in each, and she has crossed out two of the circles.)

Ms. Arenas: Y ¿como supiste?

(And how did you know?)

Delia: Porque los conté, conté cuántos huevitos quedaron.

(Because I counted them, I counted how many eggs were left.)

(Delia points to the six leftover eggs that she counted.)

Immediately after Delia's explanation, Yessenia volunteered to share her solution. Yessenia's contribution was significant both because she was a very shy student who rarely volunteered to share and because she presented a very articulate explanation describing how she solved the problem (see fig. 2.7). She grounded her explanation in the story's context, and her opportunities to hear peers, including Delia, explain their solutions may have supported her explanation.

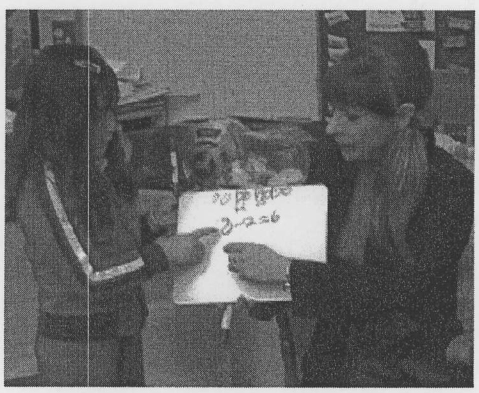

Fig. 2.7. Yessenia explains her solution to a multistep problem.

Ms. Arenas: A ver Yessenia, explícanos.

(Let's see; Yessenia, tell us how you did it.)

Yessenia: Ud, tenía dos cajitas, y tenía cuatro huevos en cada caja.

(You had two boxes, and you had four eggs in each box.)

Ms. Arenas: Y eran ocho, ¿verdad?

(And it was eight, right?)

(Ms. Arenas points to the number 8 that Yessenia has written on her whiteboard.)

Yessenia: Eran ocho. Y después comió dos y puse una linea, y luego conté y quedaron seis. Comió dos y quedaron seis.

(There were eight. And then you ate two and I put a line, and I counted and there were six left. You ate two, and there were six left.)

(Yessenia points to drawing. She drew eight circles and then a line to separate two of the circles—the eggs that Ms. Arenas ate—from the rest of the set.)

Ms. Arenas: Y aquí, ¿qué pusiste?

(And here, what did you put here?)

(Ms. Arenas points to number sentence that Yessenia has recorded, $8 - 2 = 6$.)

Yessenia: Ocho menos dos igual a seis.

(Eight minus two equals six.)

The fact that children in kindergarten struggle to articulate their mathematical reasoning is not surprising. Even older students find challenging the need to clearly explain ideas that they are in the process of learning. Of significance here is how stories, which are familiar to children both in their form and—in the stories told by Ms. Field and Ms. Arenas—in their content, provided a structure that guided young children while they explained the steps that they took to solve a given problem.

Impact on Students' Learning: Posttest Results

The previous sections have described how two teachers used stories both to generate mathematical problems and to support students while they solved problems and discussed their thinking. To further document the effect of these instructional strategies on students' learning, we conclude with a brief overview of students' performance on an end-of-the-year problem-solving assessment that we administered to all students ($n = 32$) in both classrooms. The assessment consisted of ten word problems, including basic join and separate problems (table 2.2, problems a and b), as well as such more-complex problems as multistep problems and division problems with a remainder (problems c and d). The teacher presented all problems orally in the students' dominant language, and students had access to various tools to support their reasoning.

Much like the students in the study by Carpenter and others (1993), the students in the classes of Ms. Field and Ms. Arenas demonstrated remarkable capabilities on the end-of-the-year assessment. Most of the

Table 2.2
Sample Postassessment Items

Problem Type		Problem
Join (result unknown)	a.	Julio had 6 cookies, and then his sister gave him 6 more cookies. How many cookies does Julio have now?
Separate (result unknown)	b.	Paola had 13 candies, and then she ate 5 of them. How many candies does she have left?
Multistep	c.	Javier had 2 bags of marbles. There were 4 marbles in each bag. Then he gave away 3 of his marbles. How many marbles does he have left?
Division with remainder	d.	Fifteen children want to paint. They are going to sit at tables, but only 4 children can fit at each table. How many tables are they going to need so that all the children can paint?

students successfully solved the most basic join and separate problems (80 percent and 75 percent, respectively); 60 percent of the students correctly solved the multiplication problem (3 × 6), 50 percent solved the division problem (15 ÷ 3), and 50 percent solved the multistep problem. The most difficult problem for students was the division problem that involved a remainder; and even so, one-third of the students solved it accurately.

When we analyzed the performance of individual students, we found that twenty-one of the thirty-two students (65.6 percent) successfully solved at least half the items on the posttest and that nine students (28 percent) solved all, or all but one or two, of the ten items correctly. Moreover, whereas students' explanations at the beginning of the year were often vague and incomplete, most students produced clear and more mathematical explanations on the posttest. Considering that many students began the year with limited counting and problem-solving skills and that some students lacked such basic skills as one-to-one correspondence and rote counting, these results are significant and indicate the power of teachers' use of "story" as a way to organize problem-solving sessions and support Latino and Latina kindergarten students as they make sense of, represent, and communicate their reasoning about basic word problems.

Conclusion

We conclude with a final episode from Ms. Field's classroom. At the end of one problem-solving session, Verónica (an ELL student) stopped Ms. Field as she was about to transition to the next activity and announced to the group, "Ms. Field, wait, let's try another one! Let's try this one. One girl have [sic] ten rings and then one [other] girl take[s] three. So how many is left?" Ms. Field quickly changed her plans and engaged students in solving and discussing Verónica's problem. Verónica's spontaneous problem posing generated a burst of activity because other students also wanted to pose mathematical stories for the class to solve. We end with this example because we believe that it captures the generative power of framing problem solving as "telling mathematical stories" for this diverse group of kindergarten students. Not only did the teachers' use of stories help students bridge home and family experiences with more formal school-based mathematics and help students learn to communicate their mathematical thinking while they also developed problem-solving and basic number skills, but the familiar narrative tone of "telling math stories" coupled with opportunities to coconstruct mathematical stories as a class created an entry point for students to pose their own mathematical problems. In short, framing, telling, and investigating mathematical stories created opportunities for all students to develop important problem-solving skills. Students ended the year not only as competent and more confident problem solvers but also as problem posers, which is no small feat for children in kindergarten.

The familiar narrative tone of "telling math stories" coupled with opportunities to coconstruct mathematical stories as a class created an entry point for students to pose their own mathematical problems.

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