"A Problem Is Something You Don't Want to Have": Problem Solving by Kindergartners

Although Adrian, when asked by his teacher what a problem is, was adamant that problems are to be avoided, educators believe problem solving is central to mathematics teaching and learning (NCTM 2000). Problem solving supports students as they apply their skills and their knowledge of mathematical concepts and processes to a range of different contexts and as they construct knowledge by reflecting on their own physical and mental actions. When children solve problems together, learning is a social process in which they learn not only from the teacher but also by discussing, arguing, and negotiating with their peers.

In his book *How to Solve It,* first published in 1945, Pólya proposed four fundamental steps for solving problems: (1) understand the problem, (2) devise a plan, (3) carry out the plan, and (4) look back and check the solution. Although he addressed complex problems, these steps are applicable to even simple word problems. Understanding the problem requires children—

- to be able to read the problem;
- to comprehend the quantities and relationships in the problem;
- to translate this information into a mathematical form; and
- to check whether their answer is reasonable.

In a study of the errors children make when solving word problems, Newman (1983) found that almost half the errors made by a large sample of sixth-grade students involved their inability to comprehend the problem or translate it into a mathematical form rather than to carry out the mathematics.

The recent emphasis on the importance of students' social interactions in their constructions of mathematics, combined with Pólya's fundamental steps, would seem to have implications for teaching problem solving to young children. Children need to discuss different interpretations of problems and the specific mathematical terminology they encounter if they are to understand the problem. To devise and then carry out a plan, children, individually or in small groups, need to identify numerical quantities and relationships, as well as discuss how to represent them in a mathematical form. Children can then attempt to solve the problem, and perhaps refine and modify their plan. Finally, young children need to be encouraged to reflect on their solution; they might be asked to explain to the teacher or their classmates why their solution makes sense or how they have represented the problem.

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Giving children opportunities to pose as well as solve problems allows them to engage with significant mathematical ideas (English 1997). Problem posing is an open-ended activity that can be a powerful assessment tool because it reveals children's understanding of problem structure; teachers may find that some children devise problems that involve significantly more difficult mathematics than is being taught. The contexts within which problems are situated may also affect the types of problems that young children pose; familiar contexts are easier and, therefore, may elicit more meaningful problems (Lowrie 2002).

Both the problems that children pose and their representations of problem situations provide the teacher with a window into children's thinking. The step of representing the problem is fundamental because representations indicate children's structuring of fundamental ideas (Lopez-Real and Veloo 1993). Young children may represent numerical quantities and the relationships among these quantities in a word problem by physically modeling the situation with concrete materials, by drawing their solution strategy, or by writing a symbolic expression. At present, however, we know little about the ways in which children's internal representations of problem situations generate external representations, such as drawings or models, during problem solving, especially for very young children. We also do not know what factors influence the development of children's representations or what the teacher's role is in supporting this process.

This article describes the results of an action research project in which young children in one classroom were encouraged to solve and pose their own problems in their first year at school.

The Kindergarten Class

The children in this class were all 5 years of age and were in kindergarten in a school in Sydney, Australia. The kindergarten class was separated into four mathematics groups on the basis of ability at the beginning of term 2—schooling is based on a four-term year—although these groupings were not fixed and some children later changed groups. The groups rotated through different activities over the week; the teacher worked with the problem-solving group while volunteer parents worked with the other three groups. The children participated in a problem-solving session once a week for sixteen school weeks, and during mathematics activity time they solved and later posed problems (see **fig. 1**). The problems involved repetition of a number pattern and included addition, subtraction, multiplication, division, and simple fractions (one-half).

The contexts of the problems (see **fig. 1**) were based on either literature read in class or familiar situations, such as occurrences on the school playground. The initial impetus for solving problems began with learning the nursery rhyme "Baa, Baa, Black Sheep." Accordingly, some of the first problems the children solved involved sharing situations similar to those encountered in that poem. The teacher modeled the situation of three people and three bags of wool by using different colored cubes. She asked, "We have three bags of wool; how many would each person get?" The children replied "one," so she repeated the question for six, then nine, bags. Because the children had little difficulty with this concept, she asked them, "What if we have ten bags?" Common responses were to give the extra bag back to the sheep or to divide the wool into equal shares.

In the problem-solving sessions, the teacher began by reading the problem to the children and showing how the quantities could be modeled with cubes. Next the children modeled the situation individually. This step was crucial because the children were not yet able to read, so the colored cubes acted as a memory aid. Then children were asked to draw a picture to show how they solved the

Figure 1

The problems used in the problem-solving sessions

Week	Problems
1	Two monsters were playing, two monsters were flying, three monsters were dancing. How many monsters were at the monsters' party? Four monsters are at a party. There are eight little cakes with cherries on top. How many cakes will each monster get?
2	Grandma has baked 6 cookies. Tom, Lizzie, and David love her cookies. How many will they get each? There were 18 cookies on the plate. Tom was very hungry and ate half of them. How many cookies were left on the plate?
3	Three dinosaurs each had two mittens. How many mittens altogether? Derek the Dinosaur knitted 4 pairs of socks. How many dinosaur feet were kept nice and warm?
4	In the story *The Shopping Basket*, Jeffrey had to buy lots of things. How many things did he have to buy? Make up your own shopping list using the same number pattern.
5	In the story of *Stellaluna*, the baby fruit bat makes friends with some baby birds called Pip, Flitter, and Flap. Stellaluna found 12 bugs. She shared them with Pip. How many bugs did each get? Flitter and Flap came to join the party. Stellaluna came to join the party. Stellaluna and Pip had to share their bugs. How many bugs did each get?
6	In *Alex's Bed*, Alex rearranges his bed to make more space. Think about your bedroom and the things that are in it. Draw a plan of your bedroom. Imagine that you could have the bedroom of your dreams. Draw a plan of how it would look.
7	In *Amy's Place*, Amy discovers some possums investigating her new tree house. If there were 5 mother possums, and each had 3 babies, how many would there be altogether?
8	In the book *Bear and Bunny Grow Tomatoes*, Bear grew lots of large, juicy tomatoes and shared them with his friend Bunny. Bear gave Bunny two boxes of tomatoes. Each box had 8 tomatoes inside. How many tomatoes did Bear give Bunny?

problem. Finally, the children explained to the teacher how they had represented the quantities and relationships in their drawings. Considerable emphasis was given to showing their thinking in their drawings, and aspects of children's pictorial representations were discussed in their group session. Groups generally attempted the same problems, but some children attempted additional problems and some children required far more support than others.

After a month the teacher introduced the children to the idea of writing their own problems by asking them to make their own shopping list using a specific number pattern (6, 5, 4, 3, 2, 1) and, two weeks later, to imagine the bedroom of their dreams. From week 11 on, as well as solving given problems, the children also wrote their own problems. The dialogue in **figure 2** took place in week 5, when the teacher asked children to explain their drawings of sharing twelve bugs among the four characters (the bat Stellaluna and the birds Pip, Flap, and Flitter) in the book *Stellaluna* (Cannon 1993). When the teacher asked other children to explain their thinking so that she could understand what they had done (see **fig. 2**), the other children often listened and watched. These explanations linked the models that the children had made with cubes, their drawings, and the method they had used to solve the problem.

The children's drawings developed markedly over the sixteen weeks. None of the children's initial drawings were structured; however, the later drawings of all children showed evidence of their solution strategies, and most children's representa-

Week	Problems
9	Creepy Crawly Caterpillar had 16 legs to walk with. After he came out of the stone prison, he only had 6 legs. How many pairs of legs did Creepy Crawly Caterpillar lose?
10	There are twelve people at Sammy's swimming party. Three are adults and the rest are children. How many people at the party are children? Eight children at the party are trying not to get sunburned noses, 5 are wearing sun cream and the rest hats. How many are wearing hats?
11	In her jewelry box, Janet has 6 gold rings. She also has 7 silver rings with diamonds. How many rings does Janet have in her jewelry box? Make up your own problem about Janet's jewelry box.
12	Students are making a beautiful rainforest mural. One afternoon in the rainforest, there were 9 flying foxes hanging from a gum tree. Some of the flying foxes flew away to look for food, leaving 4 flying foxes hanging from the tree. How many flying foxes flew away? Write your own problem about the rainforest.
13	There were twelve children in the playground at lunchtime. Seven children were playing hopscotch and the rest were skipping. How many children were skipping? Write your own problem about the school playground.
14	Geoffrey went to the zoo to see all the animals. He brought along ten slices of bread for the elephants. When he had finished feeding them, he had two slices of bread left. How many pieces of bread did the elephants eat? Write your own problem about the zoo.
15	Nine children have been listening carefully, and Mrs. S wants to give each one a sticker. When Mrs. S looks in her drawer, she has only 3 stickers left. How many more stickers does she need to buy so each good listener gets one? Here is a number sentence. You may write a problem for this number sentence, or you may write your own problem. 4 + 6 = ____.
16	In the story *Alexander, Who Used to Be Rich Last Sunday*, Alexander spent all the money that his grandparents gave him. If Alexander had twenty dollars and he bought three lollipops that cost two dollars each, how much money would he have left?

tions became increasingly organized in the way groupings were depicted. By the end of the year, all the children in the class were able to model the problems using concrete materials, and understood the concept that one cube represented one unit. **Figure 3** shows the change in one child's (Jeffrey's) representations over four months. His first drawing had no structure, but in the second problem session, he drew lines to indicate sharing cookies among people. Subsequent drawings show increasing structure; for example, by week 7 he used both size and alignment to show the five mother possums, each connected to a group of three babies. By week 10, he began to label group elements and write equations, first with assistance, then independently. By week 13, Jeffrey's drawing showed clearly separated, labeled groups, as well as the corresponding equation.

The concrete materials (cubes) that the children used to model the problems were colored, so they naturally chose color as one of their first strategies for depicting problem parameters and relationships. They subsequently developed a variety of other strategies in their drawings, however, including—

- size and pictorial details (skipping ropes, etc.);
- separation for subtraction and addition;
- crossing out and partitioning of sets for subtraction;
- drawing lines to indicate sharing relationships;
- array structure to show equal groups in a multiplicative situation; and
- letters and words to label sets or elements of sets.

Although the use of size, pictorial details, and separation to show groupings would seem to be a natural part of drawing a picture for young children, the use of letters to label group elements or words to label the groups themselves might not be predicted. One child suggested using letters to show adults and children, and this idea was adopted by a number of the other children, including Jeffrey (see **fig. 3**). The impetus for this strategy may have come from the emphasis on initial sounds in early reading. Both discussions in their groups and the children's drawings showed that the children adopted representational strategies used by others in their group.

Figure 2

Discussion of the drawings

Teacher. What do we have here? Tell me what you have drawn. It looks very effective.

Andy. Stellaluna, Pip, Flitter, and Flap, and they get three each.

Teacher. And how have you shown us which is which?

Andy. Lines and color.

Teacher. Lines and color? Very interesting. Tell me, John, what have you drawn?

John. This is Stellaluna, that's Pip, that's Flap and Flitter.

Teacher. Very good, and how can you show me whose cubes are whose? Whose insects are whose? You show me.

John. Because I drew arrows.

Teacher. And arrows are pointing at . . .

John. Three birds and Stellaluna.

Teacher. Very good. So the question said, "How many bugs did each get?"

John. Three.

Teacher. Can you write the answer for me if you know how to write the number? Let's have a look at what Laura has done in the last problem. Tell me how you have drawn it.

Laura. That's Stellaluna, that's Flap.

Teacher. That's Flitter and Pip, is that right? And how have you shown who has which insects? What have you done to show me that?

Laura. I put them behind.

Teacher. You put them behind—it looks as though those birds are laying eggs, doesn't it! I think that's a very good idea. I think that's very clear for me to understand how you were thinking. I think anybody could look at that and know exactly how you were thinking, and that's really important, isn't it, when we draw our problems? Good job.

Use of Equations

In week 10, a child in the most competent group asked how equations were written using plus signs. In response the teacher worked through an example, and children began writing equations, at first with assistance. Soon more than half the children in the group could write simple addition and subtraction equations. Those children who wrote equations tended to draw groups that were delineated in some way; that is, separated, labeled, or shown with different pictorial details (for example, stick figures wearing hats). The children first translated from a written problem to a concrete model (using cubes), then to a pictorial representation, and finally, to a symbolic form, so their writing of an equation was a mapping of the concrete or pictorial representation rather than of the written problem. The equations were clearly related to the children's representations of problem structure; for example, for the possum problem (see **fig. 1**), one child wrote $4 + 4 + 4 + 4 + 4 = 20$, whereas another separated mother possums and their babies and wrote $1 + 3 + 1 + 3 + 1 + 3 + 1 + 3 + 1 + 3 = 20$. The children may not have been able to write and solve the equations in the absence of accompanying representation, although they likely would have gradually dispensed with the intermediate representation as they gained counting skills. At this stage the children were using a count-all strategy to determine the answers to the problems.

Some of the children realized that problems could be represented in different ways. For example, the following discussion occurred for the problem in week 15 (see **fig. 1**) about Mrs. S, who wanted to give stickers to nine children but had only three stickers:

Teacher. Anthony, tell me what you've done.

Anthony. These were the 3 that were left in the drawer, and you needed to buy 6 more.

Teacher. Right, so what's your number sentence?

Anthony. 3 + 6 = 9 [followed by further conversation about Anthony's drawing].

Teacher. Jake, tell me what you've done.

Jake. These are the children, and these ones are the ones that you didn't have.

Teacher. So the ones with the crosses on and the circle round them are the stickers I had to buy.

Jake. Yes.

Teacher. Right, tell me about your number sentence.

Jake. 9 – 6 = 3.

Teacher. So you did it as a take-away. How did you know it was a take-away?

Jake. Because you didn't have any stickers. So we had to take it away.

Teacher. So they'd already been taken away, they'd already been used. That's very clever. That's interesting, because, look, Anthony did a plus number sentence for the problem and you did a take-away one. Who's right, do you think?

Jake. Both.

Teacher. Why do you think you're both right?

Jake. Because you can do it both ways.

Another child, Craig, realized that the problem about the flying foxes (in week 12; see **fig. 1**) could be represented in two different ways. He wrote two equations, 4 + 5 = 9 and 9 – 5 = 4.

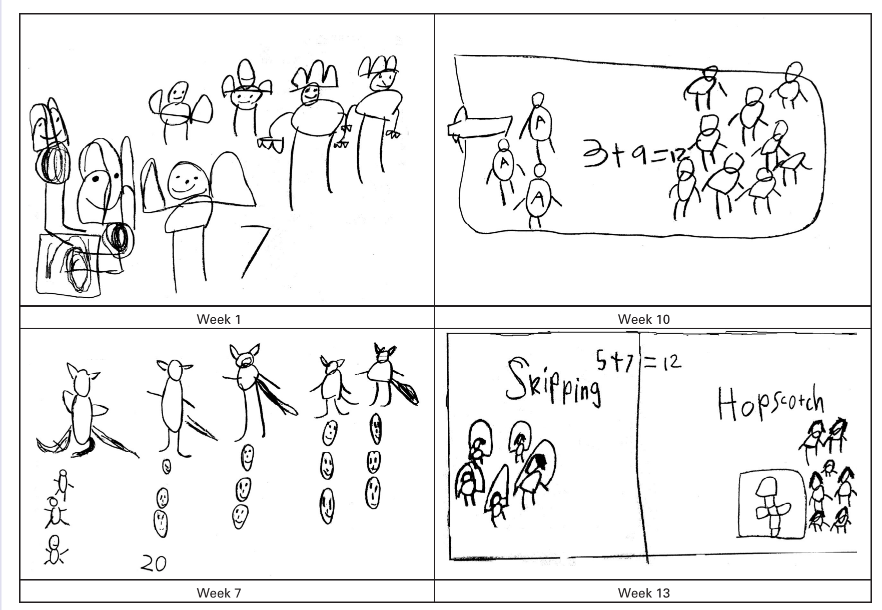
Children had difficulty when they attempted to write equations for multistep problems, such as the problem in week 16, for which they had to work out change from $20 if Alexander bought three lollipops for $2 each. The teacher did not model this problem for the group in which the children were most confident about writing equations. The children who successfully wrote equations (Craig and Kay) worked out the first step (how much the lollipops would cost) with concrete materials, marked off six dollars in their drawing, then wrote 20 – 6. Children who tried to show the relationship between dollars and lollipops in their drawing, however, became con-

Problem:

Grandma has baked 6 cookies. Tom, Lizzie, and David love her cookies. How many will each get?

Figure 3

Jeffrey's representations of four problems grew over time from having no structure to having clearly labeled groups with an equation.



fused when they tried to write the equation. They focused on the number of lollipops and wrote the following expressions: 20 + 3 = 23 or 20 – 3 = 14 and 20 – 3 = 16.

These examples show that the children were trying to understand the problem and represent its meaning in mathematical form. The teacher's emphasis on asking children to articulate their thinking meant that they were looking back at their solution strategies. At this early stage, however, devising a plan was essentially teacher-directed in its form (modeling and drawing), albeit the children's representations were individual.

Posed Problems

The week after the children began writing equations, the teacher asked them to write their own word problem about Janet's jewelry box (week 11; see **fig. 1**). All the children replicated the operation of the problem that they had just solved and, accordingly, wrote addition word problems. Usually the children modeled the structure of their self-generated problems on the structure of the given problem that immediately preceded the ones posed by the children. Exceptions did occur, however, and one of the problems posed later in the term—that about the zoo in week 14—produced a range of responses. The problems in weeks 12 and 14 were not simple structures to model, in that their initial and final states were given and the children were asked to determine the change. The complexity of this form may have prompted the variety of problem types that the children generated about the zoo. Two children from the group that needed most support dictated addition problems:

- There was a giraffe, a tiger, and a monkey. How many animals were there? (Naomi)
- There was a person, a parrot, and a monkey walking at the zoo. How many were there? (Adam, with prompting)

Andrew wrote his own unique problem: "Jaguars are extinct. How many jaguars are left?" Four other children modeled the same operation as the original problem (subtraction), but the structure of their problem was simpler; instead of a change situation (10 – □ = 2), they made the result the unknown.

- There was 1 lion and 10 fairy penguins. The lion ate 2 of the penguins. How many were left? (Craig)
- There were 5 kookaburras and 1 snake. The

snake ate 2 kookaburras. How many were left? (Anthony)

- There was 1 monkey and 4 bananas. The zookeeper took away 1 banana. How many bananas can the monkey eat? (Isabella)
- There were 4 birds and 1 lion. The lion ate 1 bird. How many birds were left? (Kay)

All these children included three parameters in their problems. This structure had not occurred in earlier, separate situations, because most children had generated take-away situations (birds flying away, koalas climbing down trees, and so on), except for Craig, who dictated the following problem about a rainforest (week 11): "There was 1 Tasmanian tiger and 5 echidnas. The Tasmanian tiger ate 2 echidnas. How many echidnas were left?" Initially he included the Tasmanian tiger with the echidnas and wrote the equation as 6 – 2 = 4 but self-corrected it to 5 – 2 = 3. For the problem about the zoo in week 14, Craig wrote 10 – 2 =, then crossed out that equation and wrote 10 – 8 = 2. For his own problem about the lion and the fairy penguins, he confidently wrote 10 – 2 = 8. Anthony and Kay also had some difficulty writing equations for their problems; Anthony first wrote 2 – 3 = 3, then crossed out that equation and wrote 5 – 2 = 3, whereas Kay wrote 4 + 3 = 3, then 4 – 1 = 3. Three children wrote more complex problems (the teacher did not transcribe these problems):

- There wor 5 lin and there wor 8 tigrs tace awae too of them ho mach wor there left (David)
- There was 10 stars and 15 children ho memey mor stars do we need? (Alison)
- Threr worer five benarnas and 2 muncies. bothe of the 2 muncies aete 1 eche. How meny benarnas were left? (Jeffrey)

None of the foregoing structures had been presented to the children. David and Jeffrey both developed multistep problems: addition (David) or multiplication (Jeffrey) followed by subtraction; Alison wrote a comparison subtraction structure. These children did not draw a representation of their written problems, but Alison and David attempted to write equations, which they found very difficult to do. David wrote 5 – 8 = 12, whereas Alison first wrote 10 + 15, then crossed it out and wrote 10 – 15 = but did not know how to solve this equation.

Only one child generated a combination subtraction problem in response to the problem in week 13. Jeffrey wrote, "There were 12 children in the school. 6 of the 12 children were bad children and they were leifin there rubish on the playgrowid. How mene children were good?" and confidently wrote the equation as 6 + 6 = 12. The other children who attempted this problem wrote addition problems, for example, "There was 3 peepl plaine with the hoops and 4 peepl plaine with the scipping ropes hoomen peepl wr there? 4 + 3 = 7" (Alison).

Conclusions

The progress of these children supports the findings of Carpenter and others (1993), who showed that kindergarten children who were taught problem solving could solve a variety of quite difficult word problems. The children in this study had spent relatively little time on problem solving, yet they were remarkably successful in representing and solving complex word problems, both with concrete materials and in drawings.

The children's drawings of the problem situations show that they used a variety of strategies to represent aspects of the contexts, including showing properties of the problem elements (color, size, pictorial details); separating groups or crossing out individual elements; partitioning sets and drawing lines to indicate sharing relationships; drawing array structures to show equal groups in multiplicative situations; and using letters and words to label elements of sets or sets themselves. An important aspect of the teaching was the emphasis on drawing a representation of the solution method as the children drew and explained not only the problem quantities but also the relationships among quantities. Toward the end of the year, most children were able to write simple equations for single-step problems with a direct relation between the quantities, and some children had a good grasp of representing problems symbolically.

The teacher also reported that the children became very motivated to write their own problems; most children in the class were enthusiastic about writing and painting their own problems in free-time sessions. The steps of (a) using a familiar context, (b) reading the problem and discussing it to ensure that students understood the problem before being asked to model and draw their solution, (c) writing a number sentence, and (d) posing their own problem helped the children move toward increasingly abstract forms of representation, and the emphasis on explaining solutions and sharing strategies ensured that the different forms were meaningfully linked. Drawing their solution, in particular, seemed to act as a bridge, linking the concrete model with the symbolic number sentence, as well as providing a focus for children to explain their different ways of representing relationships.

These kindergarten children were able to represent problems symbolically, write equations to represent quantities and relationships, and generate meaningful problems—accomplishments that are rarely reported for children at such an early age. The results suggest that teachers should question their assumptions about kindergarten children's understanding of mathematics. This small study indicates that if problem solving and posing are a valued part of the teaching environment, and if the teacher scaffolds children's learning, then young children can solve and pose quite difficult problems.

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