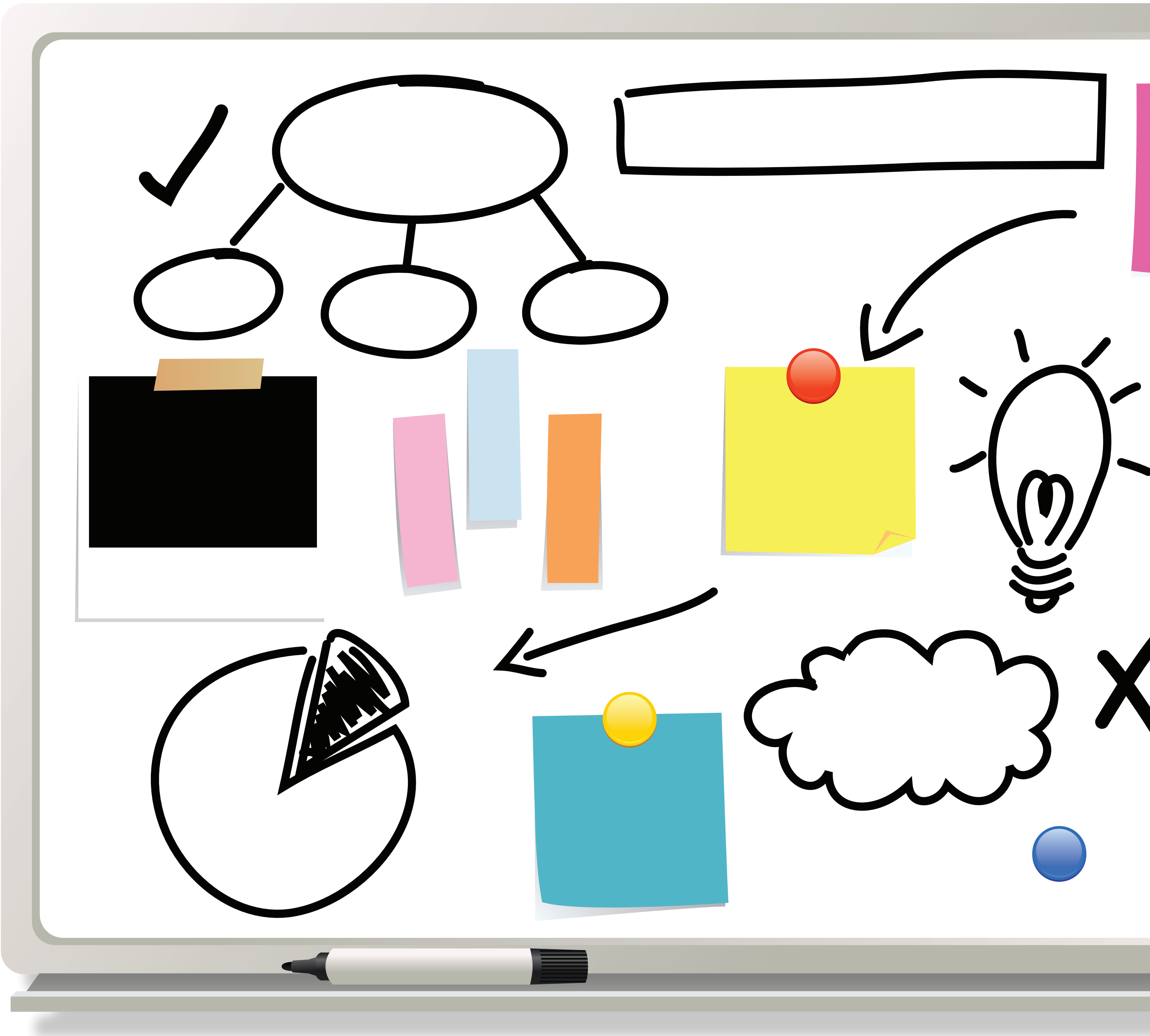

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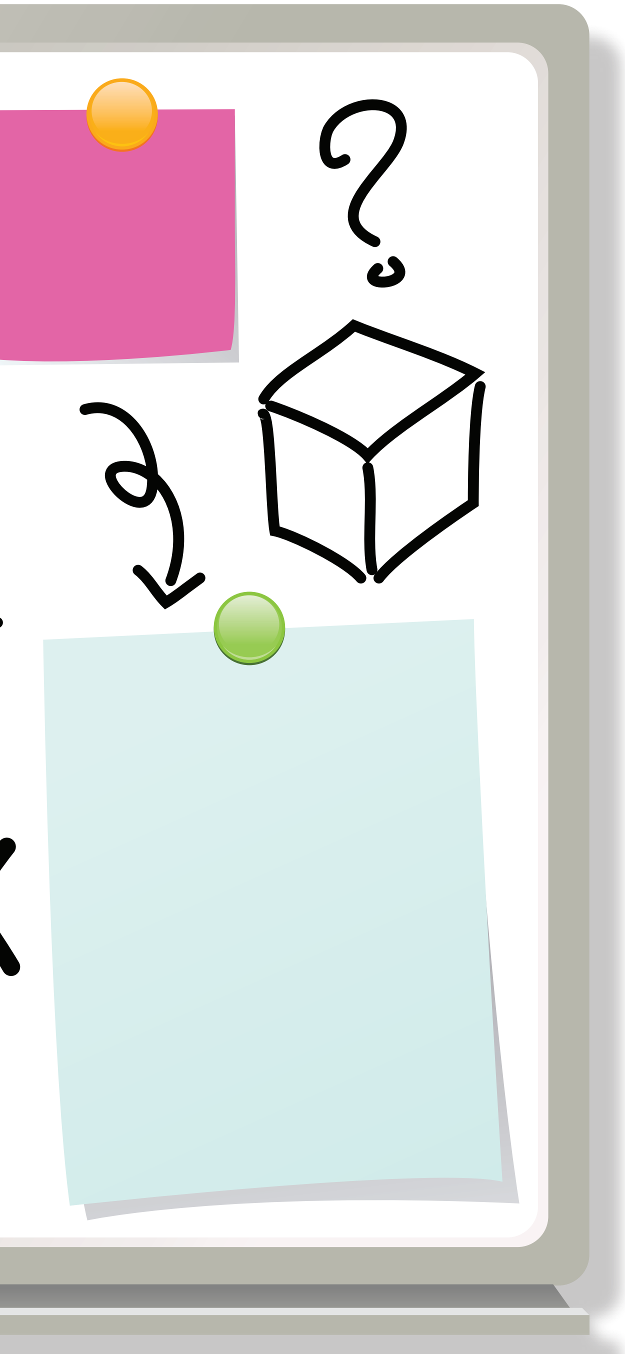
# MATHEMATICAL GRAPHIC ORGANIZERS

BY ALAN ZOLLMAN



**This alternative to standard four-step problem solving uses graphic organizers to give teachers quick, efficient diagnoses of students' individual abilities and a comfortable, familiar method to facilitate instruction.**

What is your first thought after reading the fourth-grade state assessment problem in **figure 1**? What do you suppose is a student's first thought?

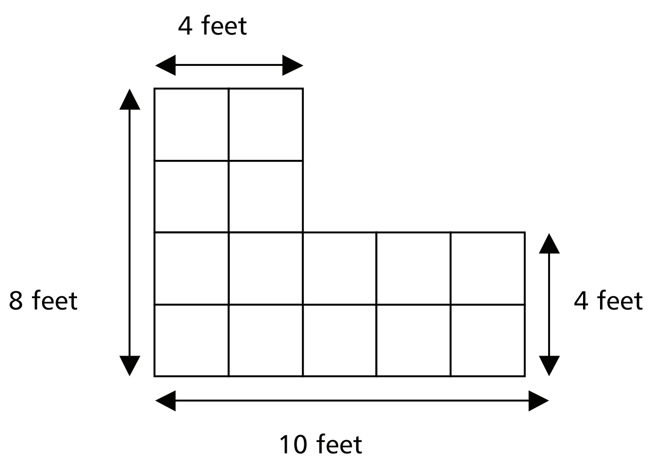
Some students initially think of counting all the squares; some think of counting the distance around the kitchen. Others think of the difference between area and perimeter or of the units (feet versus square feet) for their final answer. Some see a rectangle with a missing section; some see three separate rectangles.

These examples illustrate that thinking is not a linear activity. Often students and teachers sense—mistakenly—that mathematical problem solving must be accomplished in a certain order; that is, we must always first "identify the problem." Such an approach limits the conceptual process of problem solving to a procedural method.

**FIGURE 1**

What are students' first thoughts when they read this fourth-grade assessment problem?

Below is a drawing of a small kitchen. Find both the length of the border needed to go around the kitchen walls, and find the area of tiling needed to cover the kitchen's floor.




## Background

As part of a math-science partnership, a university mathematics educator and ten elementary school teachers developed a novel approach to mathematical problem solving derived from research on reading and writing pedagogy. Specifically, research indicates that students who use graphic organizers to arrange their ideas improve their comprehension and

## Meeting instructional goals

A major objective of elementary school math curriculum is for students to improve their problem solving (NCTM 2000, 2006) by honing these skills (NCTM 2006, p. 10):

- **Use** mathematics to solve problems.
- **Apply** logical reasoning to justify procedures and solutions.
- **Design** and analyze multiple representations to learn, make connections among, and communicate about ideas within and outside math.

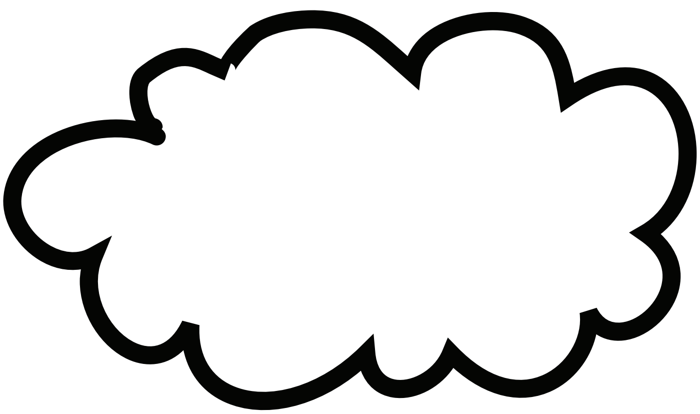
Reading and writing graphic organizers have crossover potential to achieve math goals; their spatial format allows students to do the following:

- **See** relationships between and among information and concepts.
- **Brainstorm** ideas without being concerned about correct order or solutions.
- **Record** thoughts, information, ideas, relationships, or strategies immediately to later organize, analyze, and synthesize them.

communication skills (Goeden 2002; National Reading Panel 2000). A graphic organizer is a visual representation of content classification (mind mapping), concept development (flow charts), and relationship comparisons (Venn diagrams). This article demonstrates the benefits of using modified graphic organizers with ten classes of third- through fifth-grade mathematics students.

## Benefits

Graphic organizers work well for the elementary-level reading and writing learning process (National Reading Panel 2000). In fact, graphic organizers are widely used by elementary school teachers in the writing process (Ellis 2004). The spatial format shows crossover potential for meeting mathematics instructional goals. Students can see relationships between and among information and concepts. They can brainstorm ideas without being concerned about correct order or solutions and can immediately record thoughts, information, ideas, relationships, or strategies to later organize, analyze, and synthesize their knowledge.



Students create the graphical-connection format. They do not have to process as much specific, semantic information to understand the information or problem (Ellis 2004). Visual organizer tools allow (and even expect) students to sort information as essential or nonessential, to structure information and concepts, to identify relationships between concepts, and to organize communication about an issue or problem (Zollman 2006).

Graphic organizers also aid in instruction and offer the teacher a quick, efficient diagnosis of the weaknesses and strengths in an individual student's problem-solving abilities and skills (Zollman 2006). Having a graphic representation allows a teacher to quickly assess, both formatively and summatively, each student's work regarding lacking concepts, deficient connections, weak procedures, faint justifications, or absent reflections.

## Process

As part of a classroom action-research project, ten elementary school teachers modified Gould and Gould's (1999) four-squares writing graphic organizer to make a four-corners-and-a-diamond organizer tool (see **fig. 2**) to use in a measurement unit with approximately 240 third-, fourth-, and fifth-grade students. Teachers worked with their students on using graphic organizers for short-answer, open responses to mathematical assessment problems that address the following five areas:

1. What do you need to find out?
2. What do you already know?
3. Brainstorm possible ways to solve this problem.
4. Try your ways here.
5. Which response items should you include? What did you learn from doing this problem?

Teachers may recognize Pólya's four-step problem-solving hierarchy embedded in the graphic organizer. Students often mistakenly think that problem-solving steps must be accomplished in a certain order. The mathematics graphic organizer does not imply this hierarchical procedure in the problem-solving

TABLE 1

Students helped design abbreviated rubric criteria that include levels for mathematical knowledge, strategy, and explanation.

| Score | Knowledge<br>How well did they do on the problem? | Strategy<br>How well did they plan? | Explanation<br>How well did they describe it? |
|---|---|---|---|
| 4 | They got everything correct. | They got everything planned. | They explained why they did everything. |
| 3 | They got almost everything correct. | They got almost everything planned. | They explained most of why they did things. |
| 2 | They got some of it correct. | They got some of it planned. | They explained some of why they did things. |
| 1 | They got a little of it correct. | They got a little of it planned. | They explained a little of why they did things. |
| 0 | They did not try. | They did not try to plan. | They did not try to explain. |

process. Our organizer varies from Pólya's in deployment, not in intent.

Students do not need the exact template of **figure 2**. After the first few uses, we have students fold a sheet of paper into fourths and dog-ear the inner folded corner. When they unfold the paper, they have five areas: four corner areas and the diamond in the center.

To accustom students to using graphic organizers, we place them in cooperative groups to share one large template. We present a problem and ask students to fill in what they know or "see." Having them brainstorm together (with the graphic organizer as a recording device) works well. Students notice where they have many items filled in and where they are missing information. Students observe that they can fill in different areas of a graphic organizer (in a nonhierarchical order) before having a solution to the problem. In fact, filling in the graphic organizer leads students to possible solutions and well-communicated justifications.

After some experience using the graphic organizer, we had students help design abbreviated rubric criteria that include levels for mathematical knowledge, strategy, and explanation (see **table 1**). Then we gave them a mathematical problem with a written solution (on a graphic organizer). The class assessed this written solution using the rubric and made recommendations. This process allowed students to reflect, with their teacher's guidance, on various aspects of an effective, written problem-solving solution.

## Problem-solving hierarchy (Pólya 1944)

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review and extend.

FIGURE 2

We modified Gould and Gould's (1999) four-squares writing graphic organizer to include five areas.

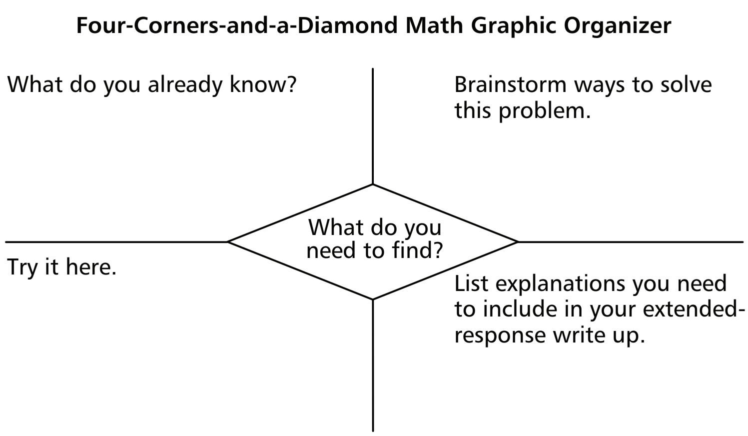
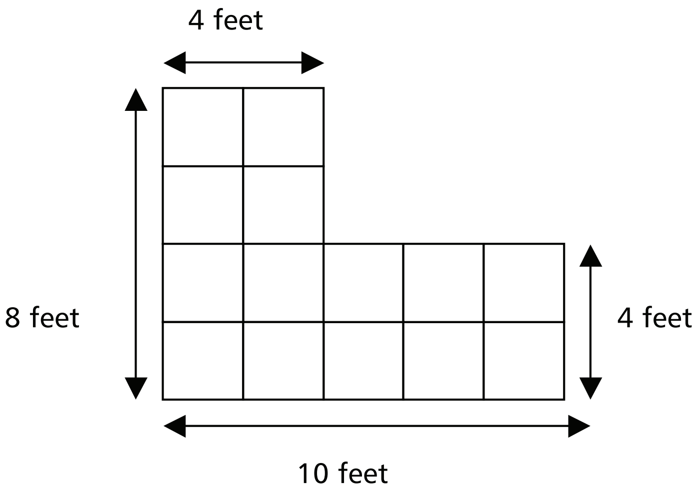


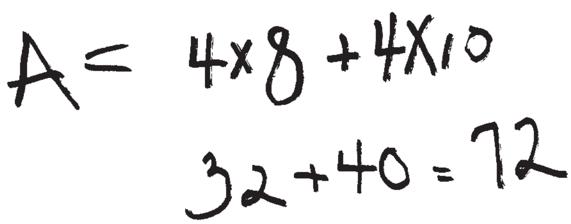
**FIGURE 3**

A student's work on the perimeter-area problem shows maturation between

(a) the preassessment sample and

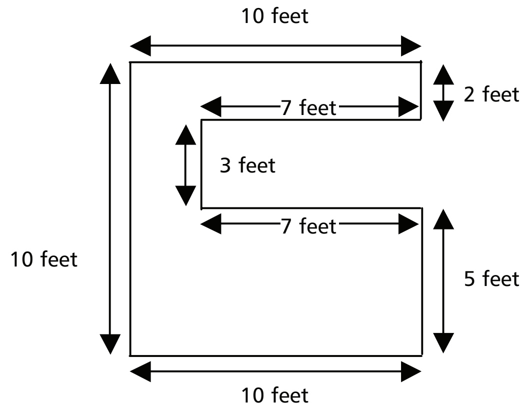
Below is a drawing of a small room. Find the area of tiling needed to cover the floor.

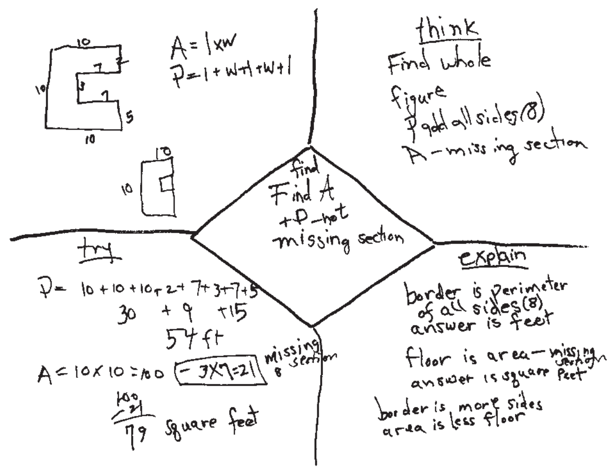



(b) the postassessment sample.

Below is a drawing of a small kitchen. Find both the length of the border needed to go around the kitchen walls, and find the area of tiling needed to cover the kitchen's floor.




## Differentiation

We found that our modified organizer works well with students at all mathematical levels. The tool provides multiple starting points for low-ability students to begin solving a problem, helps average-ability students organize their thinking strategies, and encourages high-ability students to improve their problem-solving communication skills. As evidence, note one fourth grader's maturation in **figures 3** and **4**.

## Results

Teachers in our project did identical investigations involving open-ended problems in measurement (perimeter and area problems) but used their own students as the subjects (see **table 2**). The district's elementary school mathematics coordinator first worked with the teachers in using the state's extended-response rubric in order to improve our teachers' scoring reliability. Collating all the teachers' results from the 240 students shows an increase in achievement on open-response mathematical problem solving from 22 percent to 64 percent (MSTD 2006).

Specifically, of 12-point maximum scores, the average perimeter pretest score of 6.90 in the third grade rose to 9.50 on the perimeter

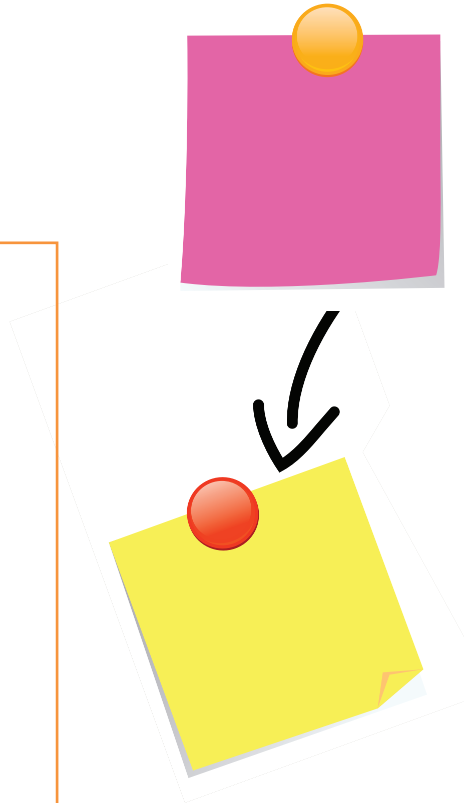
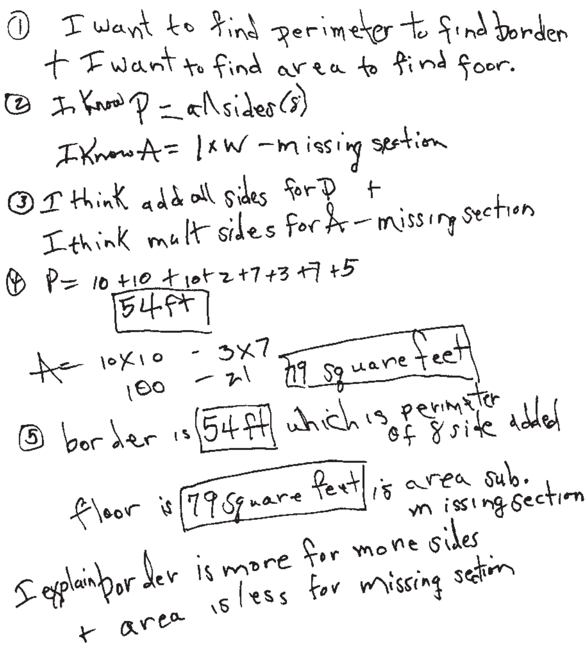


**FIGURE 4** The same student derived a final written solution from the graphic organizer.




posttest. And on the open-ended area problems, third graders' scores went from 6.30 to 9.10.

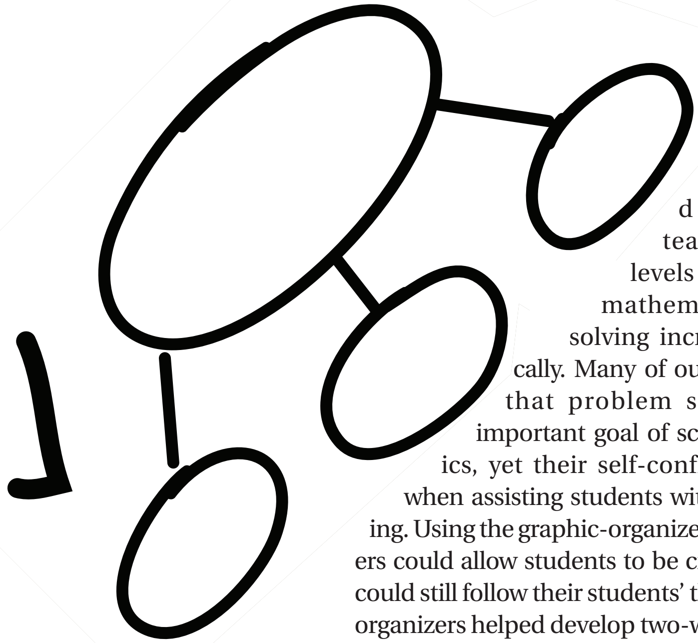
Similarly, fourth graders' pretest scores of 4.95 rose to 6.65 on the perimeter posttest. On area problems, fourth graders' pretest scores of 4.71 rose to 5.76 by the posttest.

Comparable increases also occurred with fifth graders. Their average pretest score of 4.70 rose to 6.89 on the perimeter posttest. Their average pretest score of 4.81 on the open-ended area problems rose to 7.90 on the posttest.

## Teacher benefits

Our ten teachers' individual investigations had a substantial effect on their mathematics teaching practices. Studying their own students, they found the use of graphic organizers in mathematical problem solving to be efficient and effective for students at all achievement levels. Teachers saw that students who normally would not attempt open-response problems now had partial written solutions. Students who normally did well on problems now had an efficient method of writing and communicating their thinking in logical, complete arguments. Teachers subsequently changed their instruction to include more writing in mathematical problem solving.

Another benefit of using mathematical graphic organizers was in teachers' affective



domain. Our teachers' comfort levels while teaching mathematical problem solving increased dramatically. Many of our teachers knew that problem solving was an important goal of school mathematics, yet their self-confidence was low when assisting students with problem solving. Using the graphic-organizer method, teachers could allow students to be creative, and they could still follow their students' thinking. Graphic organizers helped develop two-way communication between teachers and students.

In particular, five of our ten teachers admitted being somewhat uncomfortable with teaching problem solving before using the graphic organizers with their students (see **table 3**). Afterward, two teachers stated that they were somewhat comfortable, and eight teachers said they were now very comfortable teaching problem solving to their students.

Initially, six teachers reported that they did not feel comfortable using the state scoring rubric in assessing students' extended responses on problem solving before using the graphic organizer. Later, all ten teachers were either somewhat comfortable (three) or very comfortable (seven) scoring students' work according to the state mathematics rubric for problem solving, thus developing a strong sense of confidence in their ability to use the state scoring rubric (MSTD 2006).

TABLE 2

Teachers in our project did identical investigations involving open-ended problems in measurement but used their own students as the subjects.

| Grade and Focus | Pretest Average (12 Possible Points) | Posttest Average (12 Possible Points) |
|---|---|---|
| Grade 3 Perimeter | 6.90 | 9.50 |
| Grade 3 Area | 6.30 | 9.10 |
| Grade 4 Perimeter | 4.95 | 6.65 |
| Grade 4 Area | 4.71 | 5.76 |
| Grade 5 Perimeter | 4.70 | 6.89 |
| Grade 5 Area | 4.81 | 7.90 |

TABLE 3

Our teachers' comfort levels while teaching mathematical problem solving increased dramatically after having their students use mathematical graphic organizers.

| Teachers' Task | No. of Teachers Pretest | | | | No. of Teachers Posttest | | | |
|---|---|---|---|---|---|---|---|---|
| | NC | SU | SC | VC | NC | SU | SC | VC |
| Teaching Problem Solving | 0 | 5 | 5 | 0 | 0 | 0 | 2 | 8 |
| Using the State Scoring Rubric | 2 | 4 | 3 | 1 | 0 | 0 | 3 | 7 |

NC: Not at all Comfortable
SU: Somewhat Uncomfortable
SC: Somewhat Comfortable
VC: Very Comfortable

## Closing thoughts

We already knew from research that graphic organizers work well with elementary school students in the reading-writing process (National Reading Panel 2000). Although our data are self-reported, we have demonstrated that a good learning strategy for reading and writing is also an effective mathematics teaching method. For our mathematics students, graphic organizers have overlapping effects in connecting, communicating, justifying, and solving mathematical problems. For our mathematics teachers, graphic organizers offer quick, efficient diagnoses of individual students' problem-solving abilities, skills, strengths, and weaknesses in a comfortable, familiar, problem-solving instructional setting.

### REFERENCES


Ellis, Edwin. *What's the Big Deal about Graphic Organizers?* 2004. www.GraphicOrganizers.com.

Goeden, J. "Using Comprehension Frames (Graphic Organizers) to Impact Students' Reading Comprehension." Thesis. Black Hills State University, 2002.

Gould, Judith, and Evan Gould. *Four Square Writing Method for Grades 1–3*. Carthage, IL: Teaching and Learning Company, 1999.

Mills, G. E. *Action Research: A Guide for the Teacher Researcher.* Upper Saddle River, NJ: Merrill/Prentice Hall, 2003.

MSTD: Math & Science Teacher Development Partnership. Final report. Illinois Math and Science Partnership, funded by NCLB, Title II, Part B, US DOE, 2006.

National Reading Panel. *Teaching Children to Read: An Evidence-Based Assessment of the Scientific Research Literature on Reading and Its Implica-*

*tions for Reading Instruction.* NIH pub. no. 00-4769. Washington, DC: U.S. Department of Health and Human Services, 2000. Also available at www.nichd.nih.gov/publications/nrp/smallbook.cfm.

National Council of Teacher of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

———. *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. Reston, VA: National Council of Teachers of Mathematics, 2006.

Pólya, George. *How to Solve It.* Garden City, NY: Doubleday & Company, 1944.

Zollman, Alan. "Four-Corners Is Better than Four-Squares: Assessment Connection for Solving Mathematics Story Problems." Presented at the Annual Conference of the National Council of Teachers of Mathematics. St. Louis, MO, April 28, 2006.

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A full-sized mathematics graphic organizer template (see **fig. 2**) is appended to the online version of this article at **www.nctm.org/tcm**.

Name________________________________

# Four-Corners-and-a-Diamond Math Graphic Organizer

What do you already know?

Brainstorm ways to solve this problem.

What do you need to find?

Try it here.

List explanations you need to include in your extended-response write up.