COSA Common Core State Standards Regional Series

"Mathematics in Action"

A Statewide Regional Series for District and School Leaders of CCSS

Elementary (3-5) Mathematics Session

Locations:

October 28, 2013 - Convention Center, Pendleton
November 4, 2013 - Holiday Inn, Wilsonville
November 7, 2013 - Deschutes Expo Center, Redmond

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Track Your Progress:
Implementing the CCSSM

Shade each rectangle to show your current understanding of each learning target.

- I can design lessons focused on students learning the CCSSM.

Starting …	Getting There …	Got It!

- I can create and analyze high quality common assessments aligned to SBAC expectations.

Starting …	Getting There …	Got It!

My experience implementing the CCSSM …

What are my successes?

What are my challenges?

What Do We Expect Students To Learn?

Domains K – 5

Domain	K	1	2	3	4	5
Counting and Cardinality (CC)	✓					
Operations and Algebraic Thinking (OA)	✓	✓	✓	✓	✓	✓
Number and Operations in Base Ten (NBT)	✓	✓	✓	✓	✓	✓
Measurement and Data (MD)	✓	✓	✓	✓	✓	✓
Geometry (G)	✓	✓	✓	✓	✓	✓
Numbers and Operations-Fractions (NF)				✓	✓	✓

Content Standard Progressions

- Number yourselves 1 – 5 at each table.
- Meet with others that share your number.
- Read the K – 2 standards in your **assigned domain**.
- How are the standards **similar** across the grades?
- How are the standards **different** across the grades?
- Return to your table and share.

Operations & Algebraic Thinking 3 – 5

Grade 3	Grade 4	Grade 5
Represent and solve problems involving multiplication and division. 1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* 2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] 4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.* **Understand properties of multiplication and the relationship between multiplication and division.** 5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* *(continued on next page…)*	**Use the four operations with whole numbers to solve problems.** 1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Generate and analyze patterns.** 5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**Write and interpret numerical expressions.** 1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **Analyze patterns and relationships.** 3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Operations & Algebraic Thinking 3 – 5

Multiply and divide within 100. 7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*		

Number & Operations in Base Ten 3 – 5

Grade 3	Grade 4	Grade 5
Use place value understanding and properties of operations to perform multi-digit arithmetic. 1. Use place value understanding to round whole numbers to the nearest 10 or 100. 2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**Generalize place value understanding for multi-digit whole numbers.** 1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 3. Use place value understanding to round multi-digit whole numbers to any place. **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. 5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**Understand the place value system.** 1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 3. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 4. Use place value understanding to round decimals to any place. **Perform operations with multi-digit whole numbers and with decimals to hundredths.** 5. Fluently multiply multi-digit whole numbers using the standard algorithm. 6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number & Operations – Fractions 3 – 5

Grade 3	Grade 4	Grade 5
Develop understanding of fractions as numbers. 1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. 2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. 3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate	**Extend understanding of fraction equivalence and ordering.** 1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** 3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual	**Use equivalent fractions as a strategy to add and subtract fractions.** 1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* 2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** 3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* 4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction

simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

fraction models and equations to represent the problem.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*
   b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*
   c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement & Data 3 – 5

Grade 3	Grade 4	Grade 5
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. 1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. 2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. **Represent and interpret data.** 3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* 4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** 5. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** 1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* 2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. 3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **Represent and interpret data.** 4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	**Convert like measurement units within a given measurement system.** 1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **Represent and interpret data.** 2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** 3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. 4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

Measurement & Data 3 – 5

6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 7. Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	**Geometric measurement: understand concepts of angle and measure angles.** 5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. 6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry 3 – 5

Grade 3	Grade 4	Grade 5
Reason with shapes and their attributes. 1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. 2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	**Graph points on the coordinate plane to solve real-world and mathematical problems.** 1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). 2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Classify two-dimensional figures into categories based on their properties.** 3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* 4. Classify two-dimensional figures in a hierarchy based on properties.

Content Standard Progressions 3 – 5

Operations & Algebraic Thinking

Similarities	Differences

Numbers & Operations Base Ten

Similarities	Differences

Numbers & Operations – Fractions

Similarities	Differences

Measurement & Data

Similarities	Differences

Geometry

Similarities	Differences

The CCSS Requires Three Shifts in Mathematics

1. **Focus:** Focus strongly where the standards focus.
2. **Coherence**: *Think* across grades, and *link* to major topics
3. **Rigor:** balance *conceptual understanding*, procedural skill and *fluency*, and *application*

Rigor: Conceptual Understanding, Fluency, and Application

- Here rigor does not mean "hard problems."
- Its a balance of three fundamental components that result in deep mathematical understanding.
- There must be variety in what students are asked to produce.

Rigor Examples

- **Conceptual Understanding**:
  3.NF.1 **Understand** a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.
- **Procedural Skill and Fluency**:
  5.NBT.5 **Fluently** multiply multi-digit whole numbers using the standard algorithm.
- **Application**:
  4.OA.3 **Solve** multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

What is Fluency?

The NCTM Principle and Standards of Mathematics (2000) defines computational fluency as having **efficient**, **flexible** and **accurate** methods for computing.

Required Fluencies in K-6

Grade	Standard	Required Fluency
K	K.OA.5	Add/subtract within 5
1	1.OA.6	Add/subtract within 10
2	2.OA.2 2.NBT.5	Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100
3	3.OA.7 3.NBT.2	Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000
4	4.NBT.4	Add/subtract within 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2,3	Multi-digit division Multi-digit decimal operations

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
6. Attend to precision.

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.
5. Use appropriate tools strategically.

7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practices

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.
2. Reason abstractly and quantitatively.	Mathematically proficient third grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3. Construct viable arguments and critique the reasoning of others.	In third grade, mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilities by asking questions such as "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students require extensive opportunities to generate various mathematical representations and to both equations and story problems, and explain connections between representations as well as between representations and equations. Students should be able to use all of these representations as needed. They should evaluate their results in the context of the situation and reflect on whether the results make sense.
5. Use appropriate tools strategically.	Mathematically proficient third grader students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
6. Attend to precision.	Mathematically proficient third grader students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
7. Look for and make use of structure.	In third grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
8. Look for and express regularity in repeated reasoning.	Mathematically proficient students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Standards for Mathematical Practices

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2. Reason abstractly and quantitatively.	Mathematically proficient fourth grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others.	In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
5. Use appropriate tools strategically.	Mathematically proficient fourth grader students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6. Attend to precision.	As fourth grader students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7. Look for and make use of structure.	In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8. Look for and express regularity in repeated reasoning.	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Standards for Mathematical Practices

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	Mathematically proficient students in grade 5should solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
2. Reason abstractly and quantitatively.	Mathematically proficient students in grade 5should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others.	In fifth grade mathematical proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Mathematically proficient students in grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
5. Use appropriate tools strategically.	Mathematically proficient fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.
6. Attend to precision.	Mathematically proficient students in grade 5 continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
7. Look for and make use of structure.	In fifth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
8. Look for and express regularity in repeated reasoning.	Mathematically proficient fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Grade 3
3.NF.3 – School Day Task

Alec and Felix are brothers who go to different schools. The school day is just as long at Felix' school as at Alec's school. At Felix' school, there are 6 class periods of the same length each day. Alec's day is broken into 3 class periods of equal length.

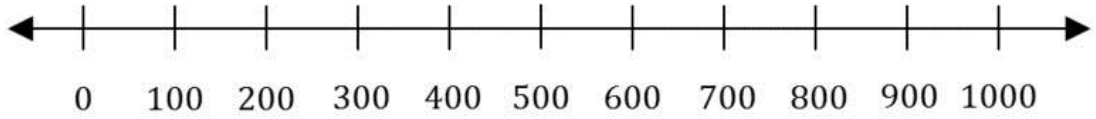
One day, it snowed a lot so both of their schools started late. Felix only had four classes and Alec only had two. Alec claims
his school day was shorter than Felix' was because he had only two classes on that day. Is he right? Explain your reasoning.

Tasks taken from www.illustrativemathemtatics.org.

Grade 4
4.NBT – Number Puzzle

1a. Find a number greater than 0 and less than 1,000 that:

- Is closer to 500 than 0,
  And
- Is closer to 200 than 500.



1b. There are many correct answers to this problem. Describe all of the numbers that are correct.

Tasks taken from www.illustrativemathemtatics.org.

Shifts in Classroom Practice

Read the
Shifts in Classroom Practice

Where is your math classroom on each continuum?

Elements of Effective Lesson Plan Design

- Which Lesson Components are a current strength in your lesson designs?
- Which Lesson Component is an area growth?

Where can I find tasks?

- www.illustrativemathematics.org – click on "Illustrations"
- www.k-5mathteachingresources.com
- www.insidemathematics.org
- www.ccssmath.org
- www.commoncoreconversation.com
- www.smarterbalanced.org
- https://grade2commoncoremath.wikispaces.hcpss.org/Grade+2+Home

Shifts in Classroom Practice

Shift 1: From *same instruction* toward *differentiated instruction.*

Same instruction for all students.	Differentiated instruction but same learning outcomes for all students.

Shift 2: From *students working individually* toward *a community of learners.*

Students work individually on tasks and seek feedback from teacher on reasonableness of strategies and solutions.	Community of learners where students hear, share, and judge reasonableness of strategies and solutions.

Shift 3: From *mathematical authority coming from the teacher or textbook* toward *mathematical authority coming from sound student reasoning.*

Correctness of solutions is determined by seeking input from teacher or textbook.	Correctness of solution is based on reasoning about the accuracy of the solution strategy.

Shift 4: From *teacher demonstrating 'how to'* toward *teacher communicating 'expectations' for learning.*

Teacher demonstrates the way in which to solve a problem and then helps students in solving it that way.	Teacher facilitates high-level performance by sharing learning goals and expectations for products that demonstrate learning.

Shift 5: From *content taught in isolation* toward *content connected to prior knowledge.*

Content presented independent of its connections to what has been previously learned.	Content presented in ways where explicit attention is given to making connections among mathematical ideas.

Shift 6: From *focus on correct answer* toward *focus on explanation and understanding*.

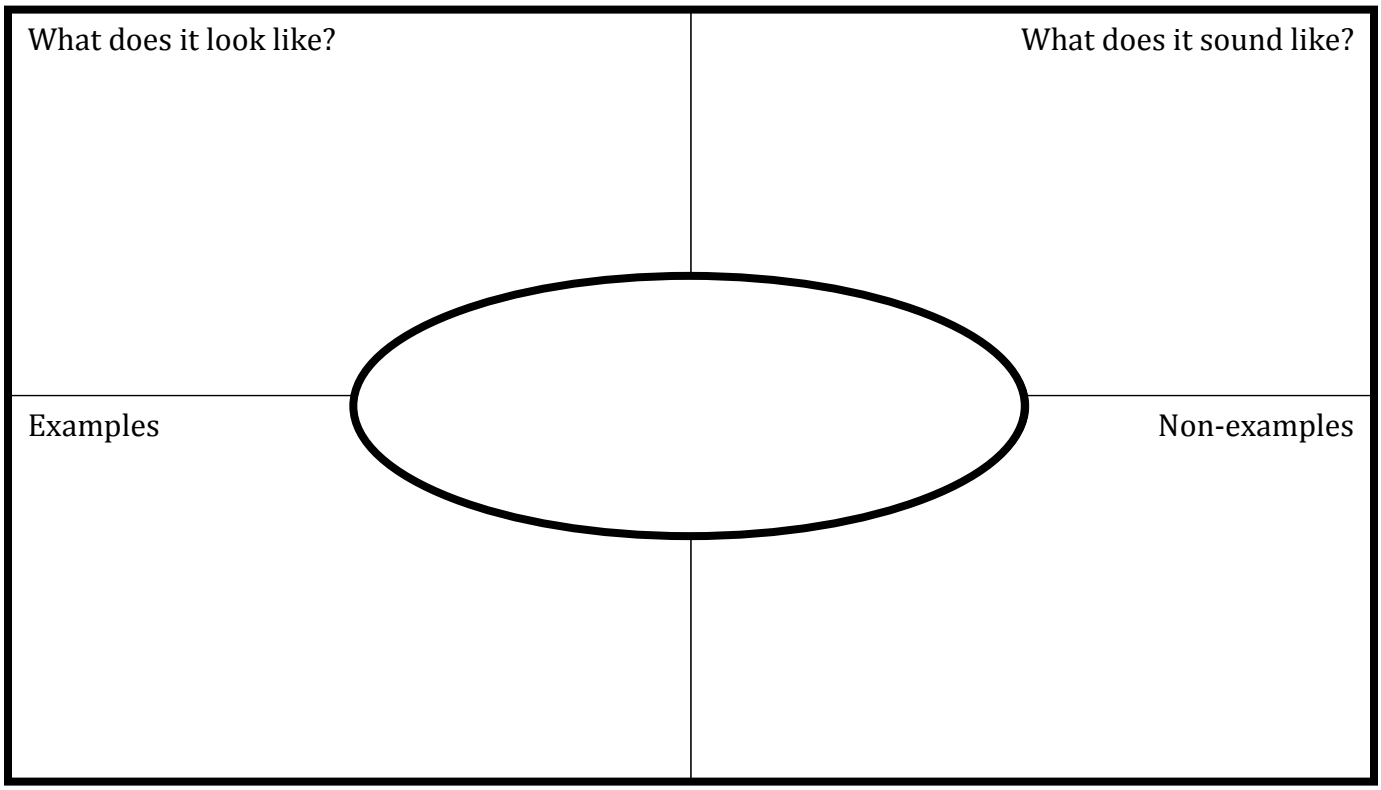
Discussions and classroom routines focus on student explanation of how they solved a task and if it is correct.	Discussions and classroom routines focus on student explanations that address why an answer is (or isn't) correct.

Shift 7: From *mathematics-made-easy for students* toward *engaging students in productive struggle.*

Mathematics is presented in small chunks and help is provided so that students reach solutions quickly and without higher level thinking.	Teacher poses tasks and challenges students to persevere and attempt multiple approaches to solving problems.

Bay-Williams, J. M., McGatha, M. B., Kobett, B. M. & Wray, J. A. *Mathematics Coaching: Resources and Tools for Coaches and Leaders, K – 12.* New York, NY: Pearson, 2013

Shifts in Classroom Practice



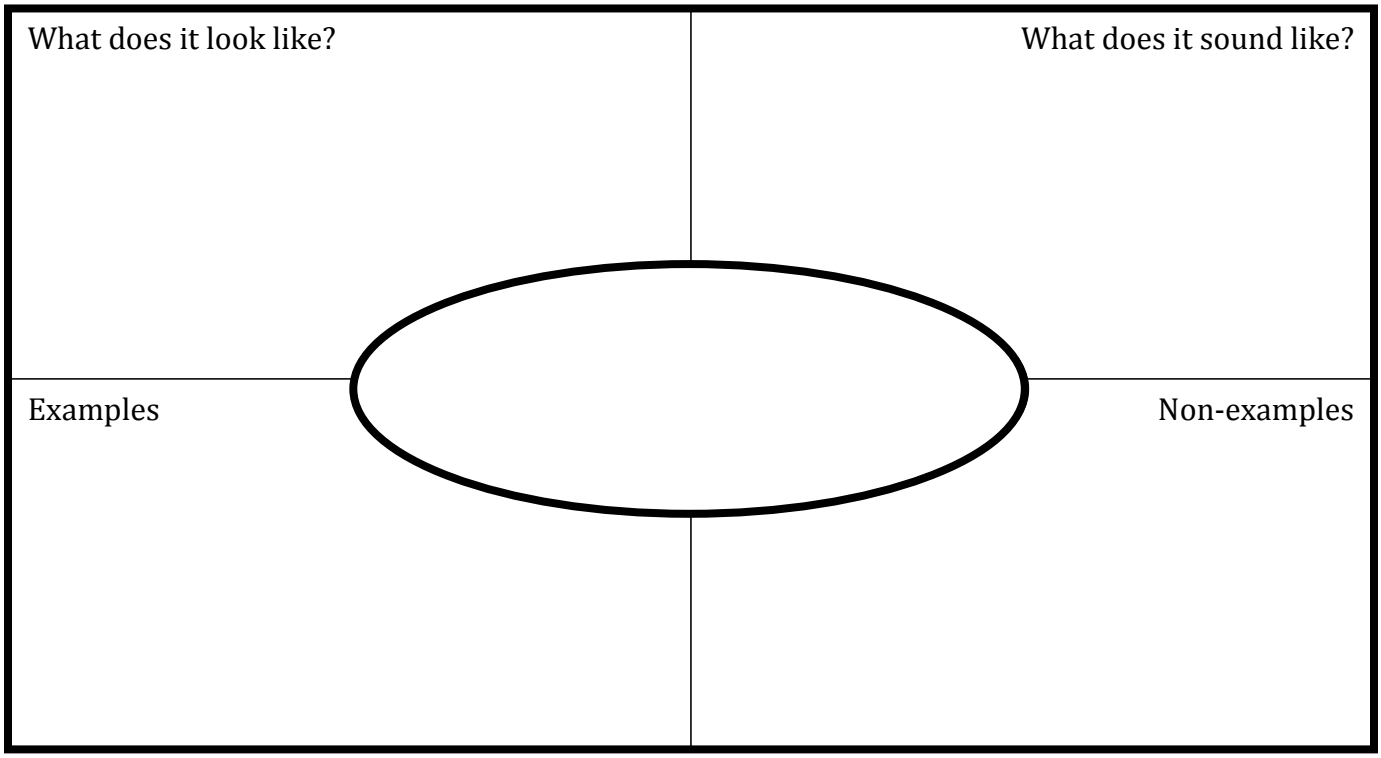


Table 2.1: Elements of an Effective Mathematics Classroom Lesson Design

	Probing Questions for Effective Lesson Design	Reflection
1. Lesson Context: Learning Targets **Procedural Fluency *and* Conceptual Understanding Balancing**	What is the learning target for the lesson? How does it connect to the bigger focus of the unit?	
	What evidence will be used to determine the level of student learning of the target?	
	Are conceptual understanding and procedural fluency appropriately balanced?	
	How will you formatively assess student conceptual understanding of the mathematics concepts *and* of the procedural skill?	
	What meaningful application or model can you use?	
	Which CCSS Mathematical Practices will be emphasized during this lesson?	
2. Lesson Process: High-Cognitive-Demand Tasks **Planning Student Discourse and Engagement**	What tasks will be used that create an a-ha student moment and leave "mathematical residue" (insights into the mathematical structure of concepts) regardless of content type at a high-cognitive-demand level?	
	How will you ensure the task is accessible to all students while still maintaining a high cognitive demand for students?	
	What strategic mathematical tools will be used during the lesson?	

	Probing Questions for Effective Lesson Design	Reflection
2. Lesson Process: High-Cognitive-Demand Tasks *(continued)*	How will each lesson *example* be presented and sequenced to build mathematical reasoning connected to prior student knowledge?	
	What are the assessing and advancing questions you might ask during guided, independent, or group practice? What are anticipated student responses to the examples or tasks?	
	How might technology and student attention to precision play a role in the student lesson experience?	
3. Introduction, Daily Review, and Closure	What activity will be used to immediately engage students at the beginning of the class period?	
	How can the daily review be used to provide brief, meaningful feedback on homework? (Five minutes maximum)	
	How will the students summarize the lesson learning targets and the key vocabulary words?	
4. Homework	How does the homework assignment provide variety and meaning to the students—including long-term review and questions—that balance procedural fluency with conceptual understanding?	

Figure 2.12:
CCSS Mathematical Practices Lesson-Planning Tool

Unit:	Date:	Lesson:
Learning target: As a result of today's class, students will be able to _____		
Formative assessment: How will students be expected to demonstrate mastery of the learning target during in-class checks for understanding?		

Probing Questions for Differentiation on Mathematical Tasks	
Assessing Questions (Create questions to scaffold instruction for students who are "stuck" during the lesson or the lesson tasks.)	**Advancing Questions** (Create questions to further learning for students who are ready to advance beyond the learning target.)

Targeted Standard for Mathematical Practice:

Which Mathematical Practice will be targeted for proficiency development during this lesson?

Tasks (Tasks can vary from lesson to lesson.)	What Will the Teacher Be Doing? (How will the teacher present and then monitor student response to the task?)	What Will the Students Be Doing? (How will students be actively engaged in each part of the lesson?)
Beginning-of-Class Routines How does the warm-up activity connect to students' prior knowledge, or how is it based on analysis of homework?		

Tasks (Tasks can vary from lesson to lesson.)	**What Will the Teacher Be Doing?** (How will the teacher present and then monitor student response to the task?)	**What Will the Students Be Doing?** (How will students be actively engaged in each part of the lesson?)
Task 1 How will the students be engaged in understanding the learning target?		
Task 2 How will the task develop student sense making and reasoning?		
Task 3 How will the task require student conjectures and communication?		
Closure How will student questions and reflections be elicited in the summary of the lesson? How will students' understanding of the learning target be determined?		

Putting it All Together

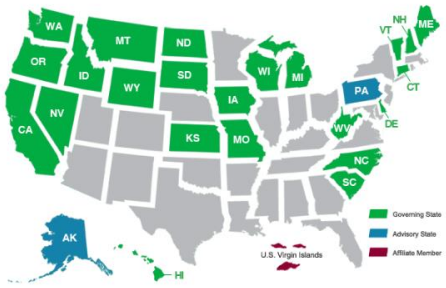
- Think of an upcoming lesson.
- What standard(s) will students learn?
- How will you know they learned it? What advancing and assessing questions will you use?
- How will you design the lesson to include tasks that incorporate the standards for mathematical practice?
- How will the lesson begin and end?

How do we know students have learned the CCSSM?

Formative vs. Summative Assessments

Formative	Summative
A process during learning	An event after learning
Descriptive feedback, use of rubrics, student self-assessment	Chapter tests, state assessment, end-of-year placement tests
Used to support ongoing growth, improvement	Used to measure achievement

SBAC Member States



SMARTER: Summative Multi-State Assessment Resources for Teachers and Educational Researchers
www.smarterbalanced.org

Four Claims Used in DRAFT SBAC Test Specifications

Claim #1 Concepts & Procedures — **40%**
Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Claim #2 Problem Solving
Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Claim #4 Modeling & Data Analysis
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

60%

Claim #3 Communicating Reasoning
Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Table 2: Estimated testing times for Smarter Balanced Summative Assessments

Test Type	Grades	CAT	Perf Task Only	Total	In-Class Activity	Total
English Language Arts/Literacy	3-5	1:30	2:00	3:30	:30	4:00
	6-8	1:30	2:00	3:30	:30	4:00
	11	2:00	2:00	4:00	:30	4:30
Mathematics	3-5	1:30	1:00	2:30	:30	3:00
	6-8	2:00	1:00	3:00	:30	3:30
	11	2:00	1:30	3:30	:30	4:00
COMBINED	3-5	3:00	3:00	6:00	1:00	7:00
	6-8	3:30	3:00	6:30	1:00	7:30
	11	4:00	3:30	7:30	1:00	8:30

Gallery Walk: SBAC

Walk around the room to read the SBAC questions and solutions.

- **What skills will students need to complete each question?**
- **How are the items scored?**

What are the **instructional** and **assessment** implications for student learning?

Performance Task – Grade 4

- Read A Trip to the Zoo Task
- Answer the questions
- Discuss the content standards and standards for mathematical practice that the task assesses. **What do students need to learn?**

Cognitive Rigor and Depth of Knowledge (DOK)

- Level 1: **Recall and Reproduction**
  Requires eliciting information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula.
- Level 2: **Basic Skills and Concepts**
  Requires the engagement of some mental processing beyond a recall of information.
- Level 3: **Strategic Thinking and Reasoning**
  Requires reasoning, planning, using evidence, and explanations of thinking.
- Level 4: **Extended Thinking**
  Requires complex reasoning, planning, developing, and thinking most likely over an extended period of time.

Sample Grade 4 Performance Task (from www.smarterbalanced.org)

A TRIP TO THE ZOO

Anna and her family go to the zoo. The zoo ticket prices, snack shop menu, and gift store prices are shown in the tables.

Zoo Ticket Prices

Type of Ticket	Price
Adult (ages 12-64)	$16
Senior (ages 65+)	$13
Child (ages 2-11)	$11
Under 2	Free

Gift Store Prices

Gift	Price
 Stuffed panda bear	$ 9
 Zoo magnet	$4
 Pack of 4 pens	$6
 Photo frame	$8

Snack Shop Menu

Food	Price
Hamburger	$5
Cheeseburger	$6
Salad	$3
Pizza	$3
Drinks	**Price**
Water	$1
Milk	$2
Juice	$3
Soda	$3

Anna's Family

- Betsy is an adult (ages 12-64)
- Grandma is a senior (ages 65 and up)
- Ray is a child (ages 2-11)
- Anna is a child (ages 2-11)

The family has $100 to spend at the zoo.

1. Use the **Zoo Ticket Prices** table and **Anna's Family** list to answer the question.

What is the total cost, in dollars, of zoo tickets for Anna's family?

2. ***Part A***

Use the **Snack Shop Menu** and **Anna's Family** list to answer the question.

Each person in Anna's family will buy one food item and one drink. Choose one food and one drink item for each person.

Enter the name for the food and drink choices for each member of the family and the total cost of the food and drink for each person.

	Food Choice	Drink Choice	Total Food and Drink Cost for Each Person
Betsy			
Grandma			
Ray			
Anna			

Part B

Use the **Snack Shop Menu** and **Anna's Family** list to answer the question.

Based on your response in Part A, what is the total cost, in dollars, of the food and drinks for Anna's family?

3. Grandma says they will spend the remaining money at the gift store.

Part A

How much money, in dollars, is remaining after the family buys zoo tickets, food, and drinks? (Remember they started with $100.)

Use the **Gift Store Prices** table to answer the question.

Part B

Anna and Ray go into the gift store. Grandma says there are 2 rules for choosing what to buy:

- Do not buy more than one of any gift.
- You must buy at least two gifts.

In your answer, you must have the following:

- Tell which gifts Anna and Ray can buy.
- Explain why there is enough money for the gifts you choose.

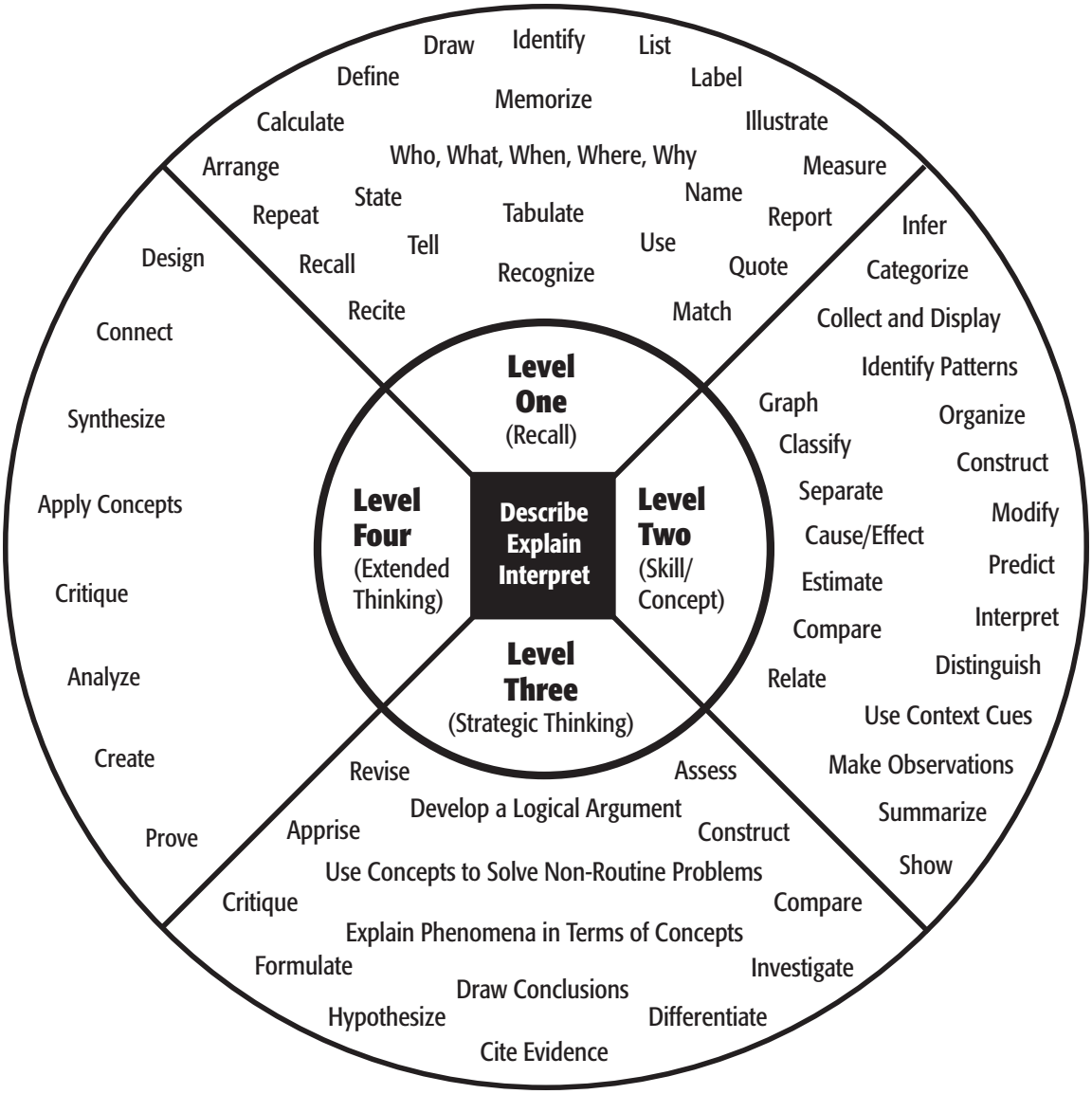
Depth of Knowledge (DOK)

Source: www.smarterbalanced.org Mathematics Content Specifications

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones & Walkup, 2009)

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	**DOK Level 1** **Recall & Reproduction**	**DOK Level 2** **Basic Skills & Concepts**	**DOK Level 3** **Strategic Thinking & Reasoning**	**DOK Level 4** **Extended Thinking**
Remember	• Recall conversations, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation.

Depth of Knowledge (DOK) Levels



Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting.	Identify and summarize the major events in a narrative.	Support ideas with details and examples.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions.
Conduct basic mathematical calculations.	Use context cues to identify the meaning of unfamiliar words.	Use voice appropriate to the purpose and audience.	Apply mathematical model to illuminate a problem or situation.
Label locations on a map.	Solve routine multiple-step problems.	Identify research questions and design investigations for a scientific problem.	Analyze and synthesize information from multiple sources.
Represent in words or diagrams a scientific concept or relationship.	Describe the cause/effect of a particular event.	Develop a scientific model for a complex situation.	Describe and illustrate how common themes are found across texts from different cultures.
Perform routine procedures like measuring length or using punctuation marks correctly.	Identify patterns in events or behavior.	Determine the author's purpose and describe how it affects the interpretation of a reading selection.	Design a mathematical model to inform and solve a practical or abstract situation.
Describe the features of a place or people.	Formulate a routine problem given data and conditions.	Apply a concept in other contexts.	
	Organize, represent and interpret data.		

Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>.

Figure 4.4:
Evaluation Tool for Assessment Instrument Quality

Assessment indicators	Description of Level 1	Requirements of the Indicator Are Not Present	Limited Requirements of This Indicator Are Present	Substantially Meets the Requirements of the Indicator	Fully Achieves the Requirements of the Indicator	Description of Level 4
Identification and emphasis on learning targets	Learning targets are unclear or absent from the assessment instrument. Too much attention is given to one target.	1	2	3	4	Clearly stated learning targets are on the assessment and connected to the assessment questions.
Visual presentation	Assessment is sloppy, disorganized, and difficult to read. There is no room for teacher feedback.	1	2	3	4	Assessment is neat, organized, easy to read, and well spaced. There is room for teacher feedback.
Time allotment	Few students can complete the assessment in the time allowed.	1	2	3	4	Test can be successfully completed in time allowed.
Clarity of directions	Directions are missing or unclear.	1	2	3	4	Directions are appropriate and clear.
Clear and appropriate scoring rubrics	Scoring rubric is either not in evidence or not appropriate for the assessment task.	1	2	3	4	Scoring rubric is clearly stated and appropriate for each problem.
Variety of assessment task formats	Assessment contains only one type of questioning strategy and no multiple choice. Calculator usage is not clear.	1	2	3	4	Test includes a variety of question types, assesses different formats, and includes calculator usage.
Question phrasing (precision)	Wording is vague or misleading. Vocabulary and precision of language is problematic for student understanding.	1	2	3	4	Vocabulary is direct, fair, and clearly understood. Students are expected to attend to precision in responses.
Balance of procedural fluency and demonstration of understanding	Test is not balanced for rigor. Emphasis is on procedural knowledge. Minimal cognitive demand for demonstration of understanding is present.	1	2	3	4	Test is balanced with product- and process-level questions. Higher-cognitive-demand and understanding tasks are present.

Sample Assessment to Analyze (NOT meant to be exemplary)

5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Name: _____ Date: _____

1. Which expressions have a value of 10? Circle all that apply. (1 point)

   A. $[9 + 1] \times 10$

   B. $100 \div 2 - 8 \times 5$

   C. $15 - 5 \times 2 - 5$

   D. $(80 + 20) \div (12 - 2)$

   E. $0.5 \times (20 - 10)$

2. Which expression is the first expression that must be simplified when evaluating $[8 + 5] \times 3 - 12 \div 4$? (1 point)

   A. $8 + 5$ B. 5×3

   C. $3 - 12$ D. $12 \div 4$

Simplify each expression. (2 points each)

3. $100 \div 10 + (9 - 1) \times 5$

4. $14 + 0.5 \times (12.5 + 7.5) - 24$

4. Write grouping symbols and/or operations (+, –, ×, ÷) to make the equation true. (2 points)

$$8 \quad 2 \quad 3 = 30$$

Analyze Assessments

- Which standards or learning targets are assessed?
- How are the mathematical practices assessed?
- Use the Evaluation of Assessment Tool to determine balance of DOK Levels, variety of assessment types, quality of questions and final product.
- How will the items be scored?
- What is proficiency?

Analyze an Assessment

- Look at the assessment.
- How does it measure against the rubric?
- How can it be improved?

What needs to be modified on your math assessments?

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