

**COSA Common Core State Standards Regional Series**  
**“Mathematics in Action”**

A Statewide Regional Series for District and School Leaders of CCSS

# **Elementary (K-2)**

# **Mathematics Session**



**Locations:**

October 28, 2013 - Convention Center, Pendleton

November 4, 2013 - Holiday Inn, Wilsonville

November 7, 2013 - Deschutes Expo Center, Redmond

**Mathematics Presenters:**

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## **Track Your Progress: Implementing the CCSSM**

Shade each rectangle to show your current understanding of each learning target.

- I can design lessons focused on students learning the CCSSM.
- I can create and analyze high quality common assessments aligned to SBAC expectations.

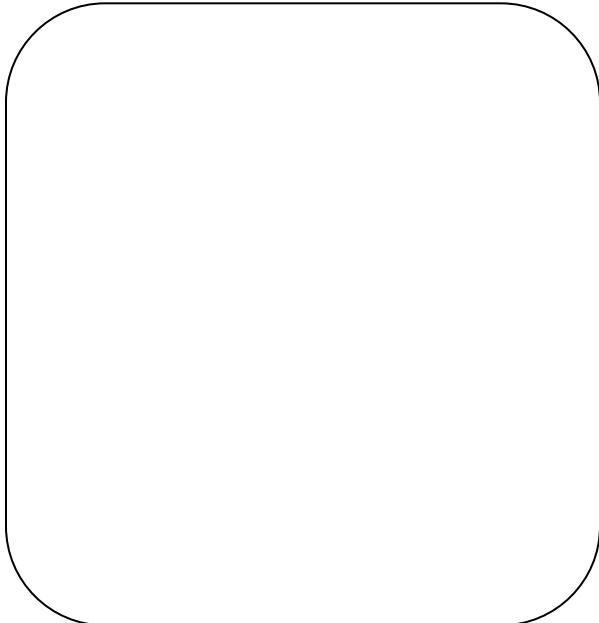
|              |                   |         |
|--------------|-------------------|---------|
| Starting ... | Getting There ... | Got It! |
|--------------|-------------------|---------|

|              |                   |         |
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| Starting ... | Getting There ... | Got It! |
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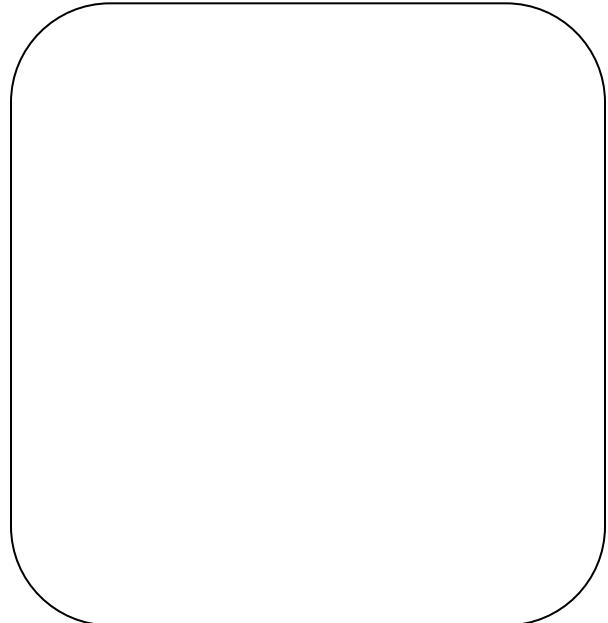
**My experience implementing the CCSSM ...**



**What are my successes?**



**What are my challenges?**





## What Do We Expect Students To Learn?

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## Domains K – 5

| Domain                                  | K | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Counting and Cardinality (CC)           | ✓ |   |   |   |   |   |
| Operations and Algebraic Thinking (OA)  | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Number and Operations in Base Ten (NBT) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Measurement and Data (MD)               | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Geometry (G)                            | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Numbers and Operations-Fractions (NF)   |   |   |   | ✓ | ✓ | ✓ |

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## Content Standard Progressions

- Number yourselves 1 – 5 at each table.
- Meet with others that share your number.
- Read the K – 2 standards in your assigned domain.
- How are the standards similar across the grades?
- How are the standards different across the grades?
- Return to your table and share.



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## Counting & Cardinality

### Kindergarten

#### Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

#### Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
  - a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name with one and only one object.
  - b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
  - c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

#### Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.<sup>1</sup>
7. Compare two numbers between 1 and 10 presented as written numerals.

| Operations & Algebraic Thinking K- 2  |  |  |
|---|--|--|
| Kindergarten  | Grade 1  | Grade 2  |
| <p><b>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</b></p> <ol style="list-style-type: none"> <li>Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.</li> <li>Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.</li> <li>Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., <math>5 = 2 + 3</math> and <math>5 = 4 + 1</math>).</li> <li>For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.</li> <li>Fluently add and subtract within 5.</li> </ol> | <p><b>Represent and solve problems involving addition and subtraction.</b></p> <ol style="list-style-type: none"> <li>Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</li> <li><b>Add and subtract within 20.</b></li> </ol> <p><b>Understand and apply properties of operations and the relationship between addition and subtraction.</b></p> <ol style="list-style-type: none"> <li>Apply properties of operations as strategies to add and subtract.<sup>3</sup> Examples: If <math>8 + 3 = 11</math> is known, then <math>3 + 8 = 11</math> is also known. (Commutative property of addition.) To add <math>2 + 6 + 4</math>, the second two numbers can be added to make a ten, so <math>2 + 6 + 4 = 2 + 10 = 12</math>. (Associative property of addition.)</li> <li>Understand subtraction as an unknown-addend problem. For example, subtract <math>10 - 8</math> by finding the number that makes 10 when added to 8.</li> <li><b>Add and subtract within 20.</b></li> <li>Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</li> </ol> | <p><b>Represent and solve problems involving addition and subtraction.</b></p> <ol style="list-style-type: none"> <li>Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</li> <li>Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.</li> </ol> <p><b>Work with equal groups of objects to gain foundations for multiplication.</b></p> <ol style="list-style-type: none"> <li>Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.</li> <li>Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.</li> <li>Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., <math>8 + 6 = 8 + 2 + 4 = 10 + 4 = 14</math>); decomposing a number leading to a ten (e.g., <math>13 - 4 = 13 - 3 - 1 = 10 - 1 = 9</math>); using the relationship between addition and subtraction (e.g., knowing that <math>8 + 4 = 12</math>, one knows <math>12 - 8 = 4</math>); and creating equivalent but easier or known sums (e.g., adding <math>6 + 7</math> by creating the known equivalent <math>6+6+1=12+1=13</math>).</li> </ol> |

## Number & Operations in Base Ten K–2

| Kindergarten   | Grade 1  | Grade 2  |
|--|--|--|
| <p><b>Understand addition as putting together Work with numbers 11–19 to gain foundations for place value.</b></p> <p>1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., <math>18 = 10 + 8</math>); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</p> <p><b>Understand place value.</b></p> <p>1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.</p> <p>2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:</p> <ol style="list-style-type: none"> <li>10 can be thought of as a bundle of ten ones — called a “ten.”</li> <li>The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.</li> <li>The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</li> <li>Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, and <math>&lt;</math>.</li> </ol> <p><b>Use place value understanding and properties of operations to add and subtract.</b></p> <p>4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p> <p>5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</p> <p>6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p> | <p><b>Extend the counting sequence.</b></p> <p>1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.</p> <p><b>Understand place value.</b></p> <p>1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:</p> <ol style="list-style-type: none"> <li>100 can be thought of as a bundle of ten tens — called a “hundred.”</li> <li>The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</li> <li>Count within 1000; skip-count by 5s, 10s, and 100s.</li> <li>Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</li> <li>Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</li> </ol> <p><b>Use place value understanding and properties of operations to add and subtract.</b></p> <p>5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>6. Add up to four two-digit numbers using strategies based on place value and properties of operations.</p> <p>7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p> <p>8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</p> <p>9. Explain why addition and subtraction strategies work, using place value and the properties of operations.</p> | <p><b>Represent and solve problems involving Understand place value.</b></p> <p>1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:</p> <ol style="list-style-type: none"> <li>100 can be thought of as a bundle of ten tens — called a “hundred.”</li> <li>The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</li> <li>Count within 1000; skip-count by 5s, 10s, and 100s.</li> <li>Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</li> <li>Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</li> </ol> <p><b>Use place value understanding and properties of operations to add and subtract.</b></p> <p>5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>6. Add up to four two-digit numbers using strategies based on place value and properties of operations.</p> <p>7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p> <p>8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</p> <p>9. Explain why addition and subtraction strategies work, using place value and the properties of operations.</p> |

## Measurement & Data K-2

| <b>Kindergarten</b>   | <b>Grade 1</b>   | <b>Grade 2</b>   |
|---|--|--|
| <p><b>Describe and compare measurable attributes.</b></p> <p>1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.</p> <p>2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. <i>For example, directly compare the heights of two children and describe one child as taller/shorter.</i></p> <p><b>Classify objects and count the number of objects in each category.</b></p> <p>3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.</p> | <p><b>Measure lengths indirectly and by iterating length units.</b></p> <p>1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.</p> <p>2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. <i>Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.</i></p> <p><b>Tell and write time.</b></p> <p>3. Tell and write time in hours and half-hours using analog and digital clocks.</p> <p><b>Represent and interpret data.</b></p> <p>4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.</p> | <p><b>Measure and estimate lengths in standard units.</b></p> <p>1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.</p> <p>2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.</p> <p>3. Estimate lengths using units of inches, feet, centimeters, and meters.</p> <p>4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.</p> <p><b>Relate addition and subtraction to length.</b></p> <p>5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.</p> <p>6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.</p> <p><b>Work with time and money.</b></p> <p>7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p> <p>8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. <i>Example: If you have 2 dimes and 3 pennies, how many cents do you have?</i></p> <p><b>Represent and interpret data.</b></p> <p>9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</p> <p>10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems<sup>4</sup> using information presented in a bar graph.</p> |

| Geometry K – 2  |   |
|---|---|
| Kindergarten  | Grade 1   |
| <p><b>Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).</b></p> <p>1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as <i>above</i>, <i>below</i>, <i>beside</i>, <i>in front of</i>, <i>behind</i>, and <i>next to</i>.</p> <p>2. Correctly name shapes regardless of their orientations or overall size.</p> <p>3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).</p> | <p><b>Reason with shapes and their attributes.</b></p> <p>1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.</p> <p>2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.<sup>4</sup></p> <p>3. Partition circles and rectangles into two and four equal shares, describe the shares using the words <i>halves</i>, <i>thirds</i>, <i>half of</i>, <i>a third of</i>, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</p> <p>4. Analyze, compare, create, and compose shapes.</p> <p>4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).</p> <p>5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.</p> <p>6. Compose simple shapes to form larger shapes. <i>For example, “Can you join these two triangles with full sides touching to make a rectangle?”</i></p> |
|   | <p><b>Grade 2</b></p> <p><b>Measure and estimate lengths in standard units.</b></p> <p><b>Reason with shapes and their attributes.</b></p> <p>1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.<sup>5</sup> Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.</p> <p>2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.</p> <p>3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words <i>halves</i>, <i>thirds</i>, <i>half of</i>, <i>a third of</i>, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</p>  |

## **Content Standard Progressions K – 2**

### **Operations & Algebraic Thinking**

| <b>Similarities</b> | <b>Differences</b> |
|---------------------|--------------------|
|                     |                    |

### **Number & Operations Base Ten**

| <b>Similarities</b> | <b>Differences</b> |
|---------------------|--------------------|
|                     |                    |

### **Measurement & Data**

| <b>Similarities</b> | <b>Differences</b> |
|---------------------|--------------------|
|                     |                    |

### **Geometry**

| <b>Similarities</b> | <b>Differences</b> |
|---------------------|--------------------|
|                     |                    |

## The CCSS Requires Three Shifts in Mathematics

1. **Focus:** Focus strongly where the standards focus.
2. **Coherence:** Think across grades, and link to major topics
3. **Rigor:** balance *conceptual understanding*, procedural skill and *fluency*, and *application*



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## Rigor: Conceptual Understanding, Fluency, and Application

- Here rigor does not mean “hard problems.”
- It’s a balance of three fundamental components that result in deep mathematical understanding.
- There must be variety in what students are asked to produce.



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## Rigor Examples

- **Conceptual Understanding:**  
1.OA.4 **Understand** subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*
- **Procedural Skill and Fluency:**  
K.OA.5 **Fluently** add and subtract within 5.
- **Application:**  
2.OA.1 Use addition and subtraction within 100 to **solve** one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

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## What is Fluency?

The NCTM Principle and Standards of Mathematics (2000) defines computational fluency as having **efficient**, **flexible** and **accurate** methods for computing.



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## Required Fluencies in K-6

| Grade | Standard          | Required Fluency  |
|-------|-------------------|---|
| K     | K.OA.5            | Add/subtract within 5   |
| 1     | 1.OA.6            | Add/subtract within 10  |
| 2     | 2.OA.2<br>2.NBT.5 | Add/subtract within 20 (know single-digit sums from memory)<br>Add/subtract within 100          |
| 3     | 3.OA.7<br>3.NBT.2 | Multiply/divide within 100 (know single-digit products from memory)<br>Add/subtract within 1000 |
| 4     | 4.NBT.4           | Add/subtract within 1,000,000   |
| 5     | 5.NBT.5           | Multi-digit multiplication  |
| 6     | 6.NS.2,3          | Multi-digit division<br>Multi-digit decimal operations  |

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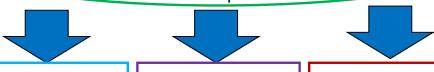
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## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



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## Standards for Mathematical Practice in Kindergarten

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

| Practice  | Explanation and Example   |
|---|---|
| 1. Make Sense and Persevere in Solving Problems.                    | <p>Mathematically proficient students in Kindergarten begin to develop effective dispositions toward problem solving. In rich settings in which informal and formal possibilities for solving problems are numerous, young children develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). Using both verbal and nonverbal means, kindergarten students begin to explain to themselves and others the meaning of a problem, look for ways to solve it, and determine if their thinking makes sense or if another strategy is needed. As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, kindergarten students begin to reason as they become more conscious of what they know and how they solve problems.</p>  |
| 2. Reason abstractly and quantitatively.                            | <p>Mathematically proficient students in Kindergarten begin to use numerals to represent specific amount (quantity). For example, a student may write the numeral “11” to represent an amount of objects counted, select the correct number card “17” to follow “16” on the calendar, or build a pile of counters depending on the number drawn. In addition, kindergarten students begin to draw pictures, manipulate objects, use diagrams or charts, etc. to express quantitative ideas such as a joining situation (Mary has 3 bears. Juanita gave her 1 more bear. How many bears does Mary have altogether?), or a separating situation (Mary had 5 bears. She gave some to Juanita. Now she has 3 bears. How many bears did Mary give Juanita?). Using the language developed through numerous joining and separating scenarios, kindergarten students begin to understand how symbols (+, -, =) are used to represent quantitative ideas in a written format.</p> |
| 3. Construct viable arguments and critique the reasoning of others. | <p>In Kindergarten, mathematically proficient students begin to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Through opportunities that encourage exploration, discovery, and discussion, kindergarten students begin to learn how to express opinions, become skillful at listening to others, describe their reasoning and respond to others' thinking and reasoning. They begin to develop the ability to reason and analyze situations as they consider questions such as, “Are you sure...?”, “Do you think that would happen all the time...?”, and ‘I wonder why...?’</p>   |
| 4. Model with mathematics.  | <p>Mathematically proficient students in Kindergarten begin to experiment with representing real-life problem situations in multiple ways such as with numbers, words (mathematical language), drawings, objects, acting out, charts, lists, and number sentences. For example, when making toothpick designs to represent the various combinations of the number “5”, the student writes the numerals for the various parts (such as “4” and “1”) or selects a number sentence that represents that particular situation (such as <math>5 = 4 + 1</math>)*.</p>  |

\*According to CCSS, “Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten in encouraged, but it is not required”. However, please note that it is not until First Grade when “Understand the meaning of the equal sign” is an expectation (1.OA.7).

|   |   |
|---|---|
| <p><b>5. Use appropriate tools strategically.</b></p>                   | <p>In Kindergarten, mathematically proficient students begin to explore various tools and use them to investigate mathematical concepts. Through multiple opportunities to examine materials, they experiment and use both concrete materials (e.g. 3-dimensional solids, connecting cubes, ten frames, number balances) and technological materials (e.g., virtual manipulatives, calculators, interactive websites) to explore mathematical concepts. Based on these experiences, they become able to decide which tools may be helpful to use depending on the problem or task. For example, when solving the problem, “There are 4 dogs in the park. 3 more dogs show up in the park. How many dogs are in the park?”, students may decide to act it out using counters and a story mat; draw a picture; or use a handful of cubes.</p> |
| <p><b>6. Attend to precision</b></p>                                    | <p>Mathematically proficient students in Kindergarten begin to express their ideas and reasoning using words. As their mathematical vocabulary increases due to exposure, modeling, and practice, kindergartners become more precise in their communication, calculations, and measurements. In all types of mathematical tasks, students begin to describe their actions and strategies more clearly, understand and use grade-level appropriate vocabulary accurately, and begin to give precise explanations and reasoning regarding their process of finding solutions. For example, a student may use color words (such as blue, green, light blue) and descriptive words (such as small, big, rough, smooth) to accurately describe how a collection of buttons is sorted.</p>  |
| <p><b>7. Look for and make use of structure</b></p>                     | <p>Mathematically proficient students in Kindergarten begin to look for patterns and structures in the number system and other areas of mathematics. For example, when searching for triangles around the room, kindergartners begin to notice that some triangles are larger than others or come in different colors- yet they are all triangles. While exploring the part-whole relationships of a number using a number balance, students begin to realize that 5 can be broken down into sub-parts, such as 4 and 1 or 4 and 2, and still remain a total of 5.</p>  |
| <p><b>8. Look for and express regularity in repeated reasoning.</b></p> | <p>In Kindergarten, mathematically proficient students begin to notice repetitive actions in geometry, counting, comparing, etc. For example, a kindergartner may notice that as the number of sides increase on a shape, a new shape is created (triangle has 3 sides, a rectangle has 4 sides, a pentagon has 5 sides, a hexagon has 6 sides). When counting out loud to 100, kindergartners may recognize the pattern 1-9 being repeated for each decade (e.g., Seventy-ONE, Seventy-TWO, Seventy-THREE... Eighty-ONE, Eighty-TWO, Eighty-THREE...). When joining one more cube to a pile, the child may realize that the new amount is the next number in the count sequence.</p>   |

## Standards for Mathematical Practice in First Grade

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

|   |  |   |  |   |  |
|---|--|---|--|---|--|
| <b>1) Make Sense and Persevere in Solving Problems.</b> | Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. They ask themselves, “Does this make sense?” First Grade students’ conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences. | <b>2) Reason abstractly and quantitatively.</b><br><br>Mathematically proficient students in First Grade recognize that a number represents a specific quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a response. For example, when solving the problem: “ <i>There are 60 children on the playground. Some children line up. There are 20 children still on the playground. How many children lined up?</i> ” first grade students may write $20 + 40 = 60$ to indicate a Think-Addition strategy. Other students may illustrate a counting-on by tens strategy by writing $20 + 10 + 10 + 10 + 10 = 60$ . The numbers and equations written illustrate the students’ thinking and the strategies used, rather than how to simply compute, and how the story is decontextualized as it is represented abstractly with symbols. | <b>3) Construct viable arguments and critique the reasoning of others.</b><br><br>Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct viable arguments about mathematics. For example, when justifying why a particular shape isn’t a square, a first grade student may hold up a picture of a rectangle, pointing to the various parts, and reason, “It can’t be a square because, even though it has 4 sides and 4 angles, the sides aren’t all the same size.” In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don’t seem to make sense. | <b>4) Model with mathematics.</b><br><br>Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed. | <b>5) Use appropriate tools strategically.</b><br><br>Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students’ conceptual understanding and higher-order thinking skills are developed. |
|---|--|---|--|---|--|

|   |  |  |
|---|--|--|
| <p><b>6) Attend to precision.</b></p> <p>Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students' ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use (<math>=, &gt;, &lt;</math>) and use clear explanations in discussions with others. For example, for the sentence <math>4 &gt; 3</math>, a proficient student who is able to attend to precision states, "Four is more than 3" rather than "The alligator eats the four. It's bigger."</p> | <p><b>7) Look for and make use of structure.</b></p> <p>Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed 'happens' to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation).</p> | <p><b>8) Look for and express regularity in repeated reasoning.</b></p> <p>Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. Thus, when solving <math>8+7+2</math>, a student may say, "I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start." Further, students use repeated reasoning while solving a task with multiple correct answers. For example, in the task "There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?" First Grade students realize that the 12 crayons could include 6 of each color (<math>6+6 = 12</math>), 7 of one color and 5 of another (<math>7+5 = 12</math>), etc. In essence, students repeatedly find numbers that add up to 12.</p> |
|---|--|--|

## **Standards for Mathematical Practice in Second Grade**

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 2 students complete.

|  |   |
|--|---|
| <b>1) Make Sense and Persevere in Solving Problems.</b>                    | Mathematically proficient students in Second Grade examine problems and tasks, can make sense of the meaning of the task and find an entry point or a way to start the task. Second Grade students also develop a foundation for problem solving strategies and become independently proficient on using those strategies to solve new tasks. In Second Grade, students' work continues to use concrete manipulatives and pictorial representations as well as mental mathematics. Second Grade students also are expected to persevere while solving tasks; that is, if students reach a point in which they are stuck, they can reexamine the task in a different way and continue to solve the task. Lastly, mathematically proficient students complete a task by asking themselves the question, "Does my answer make sense?"  |
| <b>2) Reason abstractly and quantitatively.</b>                            | Mathematically proficient students in Second Grade make sense of quantities and relationships while solving tasks. This involves two processes- decontextualizing and contextualizing. In Second Grade, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 25 children in the cafeteria and they are joined by 17 more children. How many students are in the cafeteria?" Second Grade students translate that situation into an equation, such as: $25 + 17 = \underline{\hspace{2cm}}$ and then solve the problem. Students also contextualize situations during the problem solving process. For example, while solving the task above, students can refer to the context of the task to determine that they need to subtract 19 since 19 children leave. The processes of reasoning also other areas of mathematics such as determining the length of quantities when measuring with standard units. |
| <b>3) Construct viable arguments and critique the reasoning of others.</b> | Mathematically proficient students in Second Grade accurately use definitions and previously established solutions to construct viable arguments about mathematics. During discussions about problem solving strategies, students constructively critique the strategies and reasoning of their classmates. For example, while solving 74 - 18, students may use a variety of strategies, and after working on the task, can discuss and critique each others' reasoning and strategies, citing similarities and differences between strategies.  |
| <b>4) Model with mathematics.</b>  | Mathematically proficient students in Second Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Second Grade students use concrete manipulatives and pictorial representations to provide further explanation of the equation. Likewise, Second Grade students are able to create an appropriate problem situation from an equation. For example, students are expected to create a story problem for the equation $43 + 17 = \underline{\hspace{2cm}}$ such as "There were 43 gumballs in the machine. Tom poured in 17 more gumballs. How many gumballs are now in the machine?"   |

|   |  |
|---|--|
| <p><b>5) Use appropriate tools strategically.</b></p>                   | <p>Mathematically proficient students in Second Grade have access to and use tools appropriately. These tools may include snap cubes, place value (base ten) blocks, hundreds number boards, number lines, rulers, and concrete geometric shapes (e.g., pattern blocks, 3-d solids). Students also have experiences with educational technologies, such as calculators and virtual manipulatives, which support conceptual understanding and higher-order thinking skills. During classroom instruction, students have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while measuring the length of the hallway, students can explain why a yardstick is more appropriate to use than a ruler.</p>                   |
| <p><b>6) Attend to precision.</b></p>                                   | <p>Mathematically proficient students in Second Grade are precise in their communication, calculations, and measurements. In all mathematical tasks, students in Second Grade communicate clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring an object, care is taken to line up the tool correctly in order to get an accurate measurement. During tasks involving number sense, students consider if their answer is reasonable and check their work to ensure the accuracy of solutions.</p>  |
| <p><b>7) Look for and make use of structure.</b></p>                    | <p>Mathematically proficient students in Second Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, students notice number patterns within the tens place as they connect skip count by 10s off the decade to the corresponding numbers on a 100s chart. While working in the Numbers in Base Ten domain, students work with the idea that 10 ones equals a ten, and 10 tens equals 1 hundred. In addition, Second Grade students also make use of structure when they work with subtraction as missing addend problems, such as <math>50 - 33 = \underline{\hspace{2cm}}</math> can be written as <math>33 + \underline{\hspace{2cm}} = 50</math> and can be thought of as, "How much more do I need to add to 33 to get to 50?"</p> |
| <p><b>8) Look for and express regularity in repeated reasoning.</b></p> |  |

## **Grade 1**

### **1.OA – Daisies**



Jasmine has eight daisies and three vases - one large, one medium-sized and one small.

She puts 5 daisies in the large vase, 2 in the medium vase and 1 in the small vase.

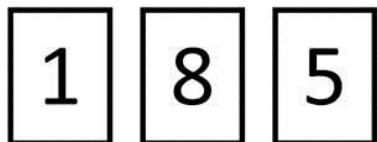
- Can you find another way to put daisies so that there are the most in the large vase and least in the small vase?
- Try to find as many ways as you can put the daisies in the vases with the most in the large vase and the least in the smallest vase. If you think you have found them all, explain how you know those are all the possibilities.

## Grade 2

### 2.NBT – Three Digit Number

Dona had cards with the numbers 0 to 9 written on them. She flipped over three of them.

Her teacher said, “*If those three numbers are the digits in another number, what is the largest three-digit number you can make?*”



- a. First Dona put the 8 in the hundreds place. Is this the right choice for the hundreds place? Explain why or why not.
  
- b. Next, Dona said, “It doesn’t matter what number I choose for the other places, because I put the biggest number in the hundreds place, and hundreds are bigger than tens and ones.” Is she correct? Explain.

## Shifts in Classroom Practice

Read the

*Shifts in Classroom Practice*



*Where is your math classroom on each continuum?*

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## Elements of Effective Lesson Plan Design

- Which Lesson Components are a current strength in your lesson designs?
- Which Lesson Component is an area growth?



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## Where can I find tasks?

- [www.illustrativemathematics.org](http://www.illustrativemathematics.org) –  click on “Illustrations”
- [www.k-5mathteachingresources.com](http://www.k-5mathteachingresources.com)
- [www.insidemathematics.org](http://www.insidemathematics.org)
- [www.ccssmath.org](http://www.ccssmath.org)
- [www.commoncoreconversation.com](http://www.commoncoreconversation.com)
- [www.smarterbalanced.org](http://www.smarterbalanced.org)
- <https://grade2commoncoremath.wikispaces.hcpss.org/Grade+2+Home>

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## Shifts in Classroom Practice

### Shift 1: From *same instruction* toward *differentiated instruction*.



### Shift 2: From *students working individually* toward *a community of learners*.



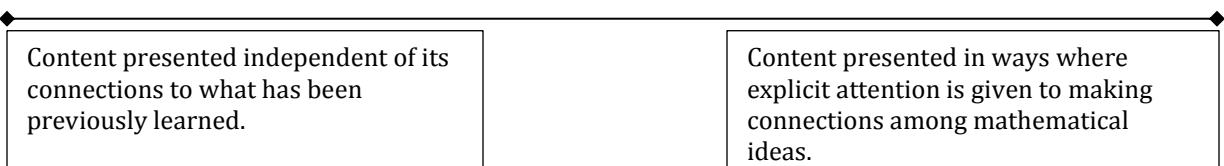
### Shift 3: From *mathematical authority coming from the teacher or textbook* toward *mathematical authority coming from sound student reasoning*.



### Shift 4: From *teacher demonstrating 'how to'* toward *teacher communicating 'expectations' for learning*.



### Shift 5: From *content taught in isolation* toward *content connected to prior knowledge*.



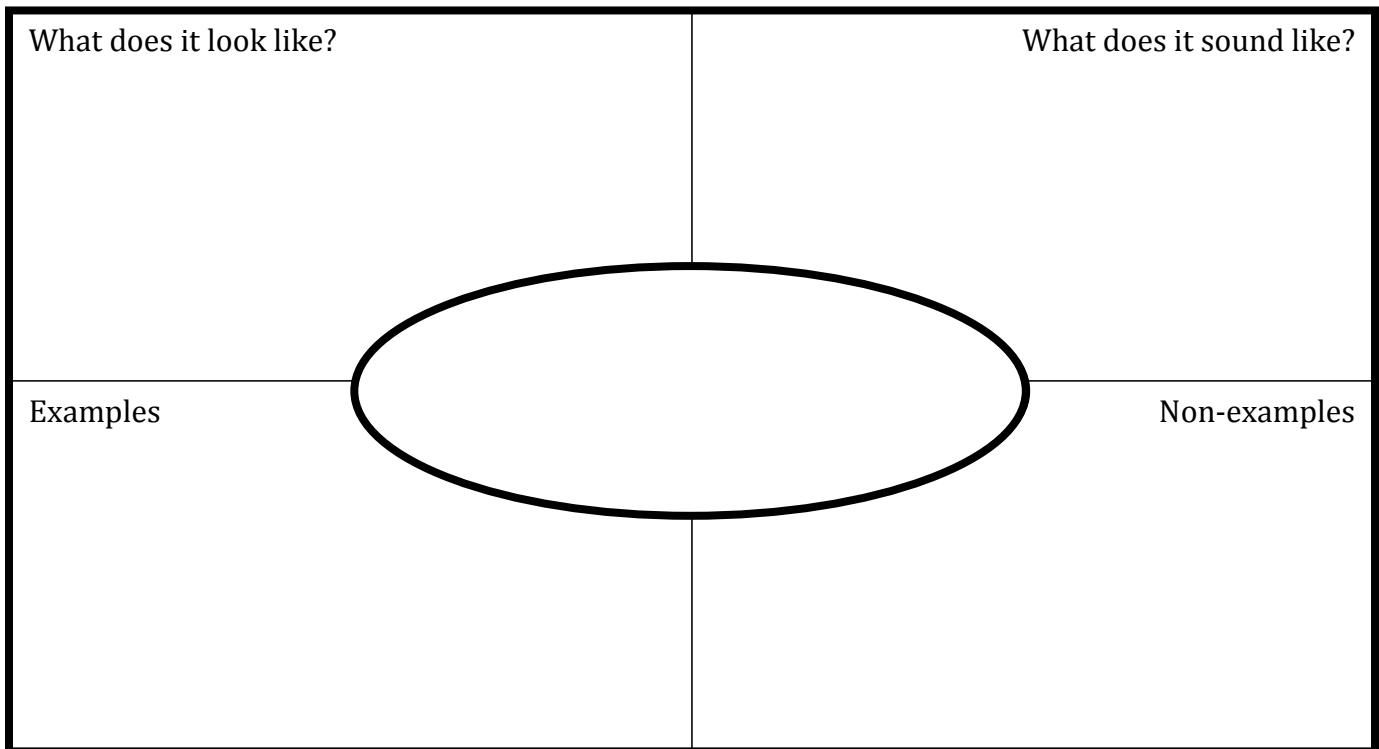
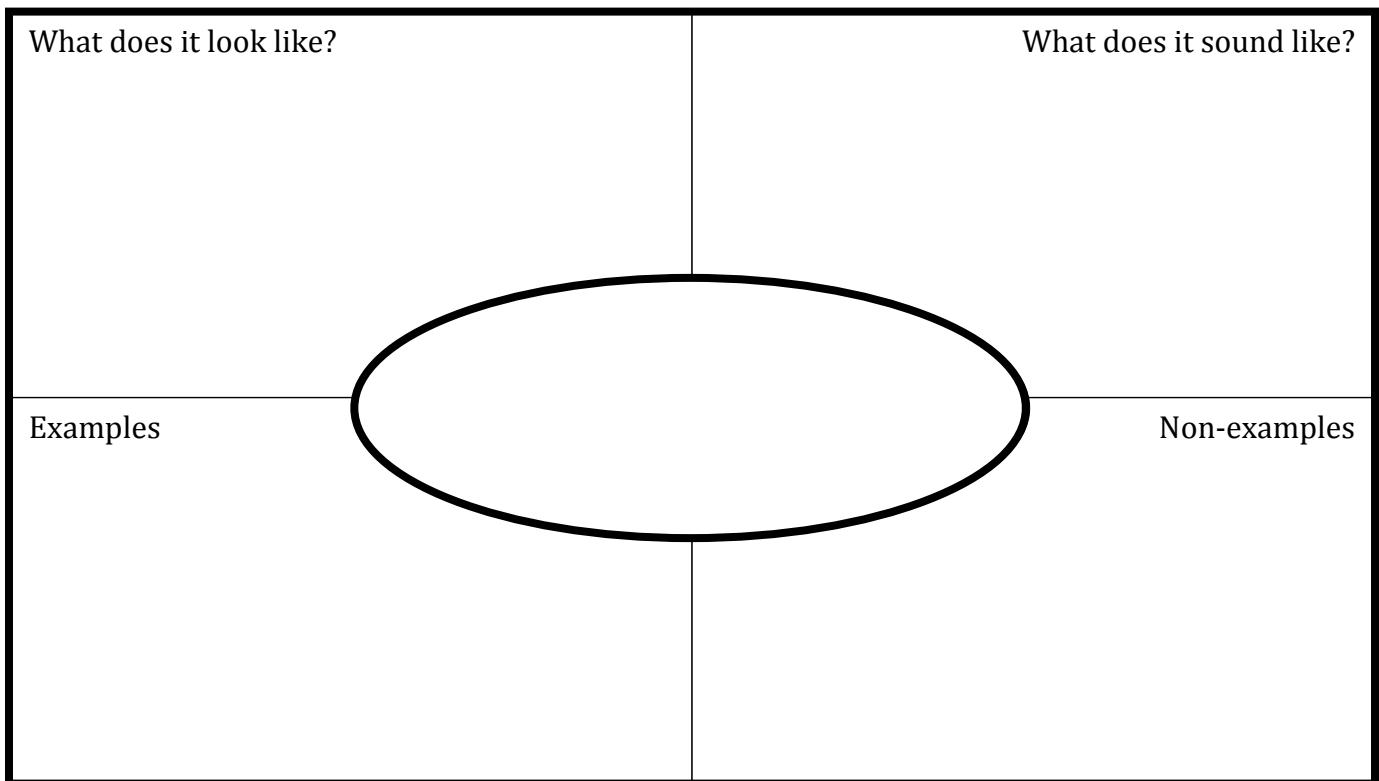
### Shift 6: From *focus on correct answer* toward *focus on explanation and understanding*.



### Shift 7: From *mathematics-made-easy for students* toward *engaging students in productive struggle*.



## Shifts in Classroom Practice



**Table 2.1:**  
**Elements of an Effective Mathematics Classroom Lesson Design**

|  | Probing Questions for Effective Lesson Design   | Reflection |
|--|---|------------|
| <b>1. Lesson Context: Learning Targets</b><br><br><b>Procedural Fluency and Conceptual Understanding Balancing</b> | What is the learning target for the lesson? How does it connect to the bigger focus of the unit?  |            |
|  | What evidence will be used to determine the level of student learning of the target?  |            |
|  | Are conceptual understanding and procedural fluency appropriately balanced?   |            |
|  | How will you formatively assess student conceptual understanding of the mathematics concepts <i>and</i> of the procedural skill?  |            |
|  | What meaningful application or model can you use?   |            |
|  | Which CCSS Mathematical Practices will be emphasized during this lesson?  |            |
| <b>2. Lesson Process: High-Cognitive-Demand Tasks</b><br><br><b>Planning Student Discourse and Engagement</b>      | What tasks will be used that create an a-ha student moment and leave “mathematical residue” (insights into the mathematical structure of concepts) regardless of content type at a high-cognitive-demand level? |            |
|  | How will you ensure the task is accessible to all students while still maintaining a high cognitive demand for students?  |            |
|  | What strategic mathematical tools will be used during the lesson?   |            |

|  | Probing Questions for Effective Lesson Design  | Reflection |
|--|--|------------|
| <b>2. Lesson Process: High-Cognitive-Demand Tasks<br/><i>(continued)</i></b> | How will each lesson <i>example</i> be presented and sequenced to build mathematical reasoning connected to prior student knowledge?   |            |
|  | What are the assessing and advancing questions you might ask during guided, independent, or group practice? What are anticipated student responses to the examples or tasks?         |            |
|  | How might technology and student attention to precision play a role in the student lesson experience?  |            |
| <b>3. Introduction, Daily Review, and Closure</b>                            | What activity will be used to immediately engage students at the beginning of the class period?  |            |
|  | How can the daily review be used to provide brief, meaningful feedback on homework? (Five minutes maximum)   |            |
|  | How will the students summarize the lesson learning targets and the key vocabulary words?  |            |
| <b>4. Homework</b>   | How does the homework assignment provide variety and meaning to the students—including long-term review and questions—that balance procedural fluency with conceptual understanding? |            |

## Figure 2.12:

### CCSS Mathematical Practices Lesson-Planning Tool

|   |  |  |
|---|--|--|
| Unit:   | Date:  | Lesson:  |
| <p><b>Learning target:</b> As a result of today's class, students will be able to _____</p> <p><b>Formative assessment:</b> How will students be expected to demonstrate mastery of the learning target during in-class checks for understanding?</p> |  |  |
| <b>Probing Questions for Differentiation on Mathematical Tasks</b>  |  |  |
| <p><b>Assessing Questions</b><br/>(Create questions to scaffold instruction for students who are “stuck” during the lesson or the lesson tasks.)</p>  | <p><b>Advancing Questions</b><br/>(Create questions to further learning for students who are ready to advance beyond the learning target.)</p> |  |
| <p><b>Targeted Standard for Mathematical Practice:</b><br/>Which Mathematical Practice will be targeted for proficiency development during this lesson?</p>   |  |  |
| <p><b>Tasks</b><br/>(Tasks can vary from lesson to lesson.)</p>   | <p><b>What Will the Teacher Be Doing?</b><br/>(How will the teacher present and then monitor student response to the task?)</p>                | <p><b>What Will the Students Be Doing?</b><br/>(How will students be actively engaged in each part of the lesson?)</p> |
| <p><b>Beginning-of-Class Routines</b><br/>How does the warm-up activity connect to students' prior knowledge, or how is it based on analysis of homework?</p>   |  |  |

| <b>Tasks</b><br>(Tasks can vary from lesson to lesson.)   | <b>What Will the Teacher Be Doing?</b><br>(How will the teacher present and then monitor student response to the task?) | <b>What Will the Students Be Doing?</b><br>(How will students be actively engaged in each part of the lesson?) |
|---|---|--|
| <b>Task 1</b><br>How will the students be engaged in understanding the learning target?   |   |  |
| <b>Task 2</b><br>How will the task develop student sense making and reasoning?  |   |  |
| <b>Task 3</b><br>How will the task require student conjectures and communication?   |   |  |
| <b>Closure</b><br>How will student questions and reflections be elicited in the summary of the lesson? How will students' understanding of the learning target be determined? |   |  |

## Putting it All Together



- Think of an upcoming lesson.
- What standard(s) will students learn?
- How will you know they learned it? What advancing and assessing questions will you use?
- How will you design the lesson to include tasks that incorporate the standards for mathematical practice?
- How will the lesson begin and end?

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## How do we know students have learned the CCSSM?

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## Formative vs. Summative Assessments

### Formative

- A process during learning
- Descriptive feedback, use of rubrics, student self-assessment
- Used to support ongoing growth, improvement

### Summative

- An event after learning
- Chapter tests, state assessment, end-of-year placement tests
- Used to measure achievement



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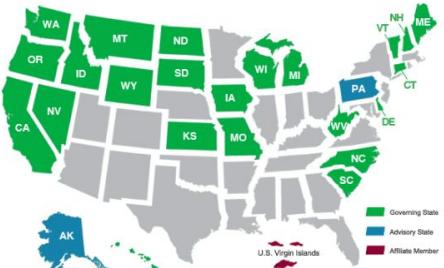
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## SBAC Member States



**SMARTER: Summative Multi-State Assessment**  
**Resources for Teachers and Educational Researchers**  
[www.smarterbalanced.org](http://www.smarterbalanced.org)

## Four Claims Used in DRAFT SBAC Test Specifications

|   |            |   |
|---|------------|---|
| <b>Claim #1</b><br>Concepts & Procedures    | <b>40%</b> | Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.                          |
| <b>Claim #2</b><br>Problem Solving          |            | Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies. |
| <b>Claim #4</b><br>Modeling & Data Analysis |            | Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.                             |
| <b>Claim #3</b><br>Communicating Reasoning  | <b>60%</b> | Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.                         |

Table 2: Estimated testing times for  
 Smarter Balanced Summative Assessments

| Test Type                      | Grades | CAT  | Perf Task Only | Total | In-Class Activity | Total |
|--------------------------------|--------|------|----------------|-------|-------------------|-------|
| English Language Arts/Literacy | 3-5    | 1:30 | 2:00           | 3:30  | :30               | 4:00  |
|                                | 6-8    | 1:30 | 2:00           | 3:30  | :30               | 4:00  |
|                                | 11     | 2:00 | 2:00           | 4:00  | :30               | 4:30  |
| Mathematics                    | 3-5    | 1:30 | 1:00           | 2:30  | :30               | 3:00  |
|                                | 6-8    | 2:00 | 1:00           | 3:00  | :30               | 3:30  |
|                                | 11     | 2:00 | 1:30           | 3:30  | :30               | 4:00  |
| COMBINED                       | 3-5    | 3:00 | 3:00           | 6:00  | 1:00              | 7:00  |
|                                | 6-8    | 3:30 | 3:00           | 6:30  | 1:00              | 7:30  |
|                                | 11     | 4:00 | 3:30           | 7:30  | 1:00              | 8:30  |

## Gallery Walk: SBAC

Walk around the room to read the SBAC questions and solutions.



- What skills will students need to complete each question?
- How are the items scored?

What are the **instructional** and **assessment** implications for student learning?

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## Performance Task – Grade 4

- Read A Trip to the Zoo Task
- Answer the questions
- Discuss the content standards and standards for mathematical practice that the task assesses. **What do students need to learn?**



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## Cognitive Rigor and Depth of Knowledge (DOK)



- **Level 1: Recall and Reproduction**  
Requires eliciting information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula.
- **Level 2: Basic Skills and Concepts**  
Requires the engagement of some mental processing beyond a recall of information.
- **Level 3: Strategic Thinking and Reasoning**  
Requires reasoning, planning, using evidence, and explanations of thinking.
- **Level 4: Extended Thinking**  
Requires complex reasoning, planning, developing, and thinking most likely over an extended period of time.

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# Sample Grade 4 Performance Task (from [www.smarterbalanced.org](http://www.smarterbalanced.org))

## A TRIP TO THE ZOO

Anna and her family go to the zoo. The zoo ticket prices, snack shop menu, and gift store prices are shown in the tables.

**Zoo Ticket Prices**

| Type of Ticket     | Price |
|--------------------|-------|
| Adult (ages 12-64) | \$16  |
| Senior (ages 65+)  | \$13  |
| Child (ages 2-11)  | \$11  |
| Under 2            | Free  |

**Snack Shop Menu**

| Food         | Price |
|--------------|-------|
| Hamburger    | \$5   |
| Cheeseburger | \$6   |
| Salad        | \$3   |
| Pizza        | \$3   |

| Drinks | Price |
|--------|-------|
| Water  | \$1   |
| Milk   | \$2   |
| Juice  | \$3   |
| Soda   | \$3   |

**Gift Store Prices**

| Gift  | Price |
|---|-------|
|    | \$9   |
| Stuffed panda bear  |       |
|   | \$4   |
| Zoo magnet  |       |
|  | \$8   |
| Photo frame   |       |
| Pack of 4 pens  |       |

### Anna's Family

- Betsy is an adult (ages 12-64)
- Grandma is a senior (ages 65 and up)
- Ray is a child (ages 2-11)
- Anna is a child (ages 2-11)

The family has \$100 to spend at the zoo.

1. Use the **Zoo Ticket Prices** table and **Anna's Family** list to answer the question.

What is the total cost, in dollars, of zoo tickets for Anna's family?

2. **Part A**

Use the **Snack Shop Menu** and **Anna's Family** list to answer the question.

Each person in Anna's family will buy one food item and one drink. Choose one food and one drink item for each person.

Enter the name for the food and drink choices for each member of the family and the total cost of the food and drink for each person.

|         | Food Choice          | Drink Choice         | Total Food and Drink Cost for Each Person |
|---------|----------------------|----------------------|---|
| Betsy   | <input type="text"/> | <input type="text"/> | <input type="text"/>                      |
| Grandma | <input type="text"/> | <input type="text"/> | <input type="text"/>                      |
| Ray     | <input type="text"/> | <input type="text"/> | <input type="text"/>                      |
| Anna    | <input type="text"/> | <input type="text"/> | <input type="text"/>                      |

**Part B**

Use the **Snack Shop Menu** and **Anna's Family** list to answer the question.

Based on your response in Part A, what is the total cost, in dollars, of the food and drinks for Anna's family?

3. Grandma says they will spend the remaining money at the gift store.

**Part A**

How much money, in dollars, is remaining after the family buys zoo tickets, food, and drinks? (Remember they started with \$100.)

Use the **Gift Store Prices** table to answer the question.

**Part B**

Anna and Ray go into the gift store. Grandma says there are 2 rules for choosing what to buy:

- Do not buy more than one of any gift.
- You must buy at least two gifts.

In your answer, you must have the following:

- Tell which gifts Anna and Ray can buy.
- Explain why there is enough money for the gifts you choose.

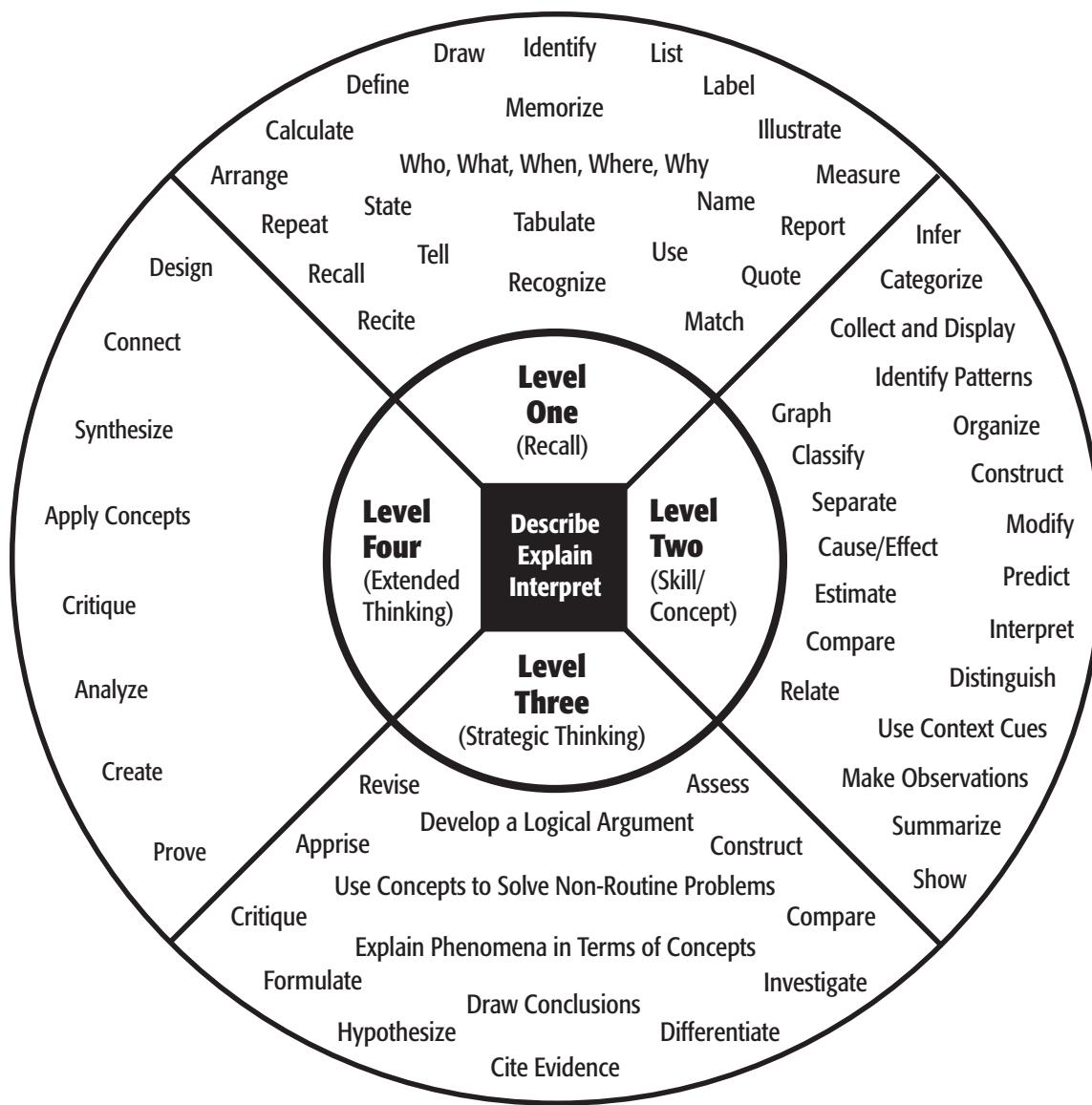
# Depth of Knowledge (DOK)

Source: [www.smarterbalanced.org Mathematics Content Specifications](http://www.smarterbalanced.org/Mathematics-Content-Specifications)

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones & Walkup, 2009)

| <b>Depth of Thinking<br/>(Webb)<br/>+ Type of Thinking (Revised Bloom)</b> | <b>DOK Level 1<br/>Recall &amp; Reproduction</b>  | <b>DOK Level 2<br/>Basic Skills &amp; Concepts</b>   | <b>DOK Level 3<br/>Strategic Thinking &amp; Reasoning</b>   | <b>DOK Level 4<br/>Extended Thinking</b>   |
|--|---|--|---|--|
| Remember   | <ul style="list-style-type: none"> <li>Recall conversations, terms, facts</li> </ul>  |  |   |  |
| Understand   | <ul style="list-style-type: none"> <li>Evaluate an expression</li> <li>Locate points on a grid or number on number line</li> <li>Solve a one-step problem</li> <li>Represent math relationships in words, pictures, or symbols</li> </ul> | <ul style="list-style-type: none"> <li>Specify, explain relationships</li> <li>Make basic inferences or logical predictions from data/observations</li> <li>Use models/diagrams to explain concepts</li> <li>Make and explain estimates</li> </ul>           | <ul style="list-style-type: none"> <li>Use concepts to solve non-routine problems</li> <li>Use supporting evidence to justify conjectures, generalize, or connect ideas</li> <li>Explain reasoning when more than one response is possible</li> <li>Explain phenomena in terms of concepts</li> </ul> | <ul style="list-style-type: none"> <li>Relate mathematical concepts to other content areas, other domains</li> <li>Develop generalizations of the results obtained and the strategies used and apply them to new problem situations</li> </ul> |
| Apply  | <ul style="list-style-type: none"> <li>Follow simple procedures</li> <li>Calculate, measure, apply a rule (e.g., rounding)</li> <li>Apply algorithm or formula</li> <li>Solve linear equations</li> <li>Make conversions</li> </ul>       | <ul style="list-style-type: none"> <li>Select a procedure and perform it</li> <li>Solve routine problem applying multiple concepts or decision points</li> <li>Retrieve information to solve a problem</li> <li>Translate between representations</li> </ul> | <ul style="list-style-type: none"> <li>Design investigation for a specific purpose or research question</li> <li>Use reasoning, planning, and supporting evidence</li> <li>Translate between problem &amp; symbolic notation when not a direct translation</li> </ul>                                 | <ul style="list-style-type: none"> <li>Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results</li> </ul>   |
| Analyze  | <ul style="list-style-type: none"> <li>Retrieve information from a table or graph to answer a question</li> <li>Identify a pattern/trend</li> </ul>   | <ul style="list-style-type: none"> <li>Categorize data, figures</li> <li>Organize, order data</li> <li>Select appropriate graph and organize &amp; display data</li> <li>Interpret data from a simple graph</li> <li>Extend a pattern</li> </ul>             | <ul style="list-style-type: none"> <li>Compare information within or across data sets or texts</li> <li>Analyze and draw conclusions from data, citing evidence</li> <li>Generalize a pattern</li> <li>Interpret data from complex graph</li> </ul>   | <ul style="list-style-type: none"> <li>Analyze multiple sources of evidence or data sets</li> </ul>  |
| Evaluate   |   |  | <ul style="list-style-type: none"> <li>Cite evidence and develop a logical argument</li> <li>Compare/contrast solution methods</li> <li>Verify reasonableness</li> </ul>  | <ul style="list-style-type: none"> <li>Apply understanding in a novel way, provide argument or justification for the new application</li> </ul>  |
| Create   | <ul style="list-style-type: none"> <li>Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept</li> </ul>   | <ul style="list-style-type: none"> <li>Generate conjectures or hypotheses based on observations or prior knowledge and experience</li> </ul>   | <ul style="list-style-type: none"> <li>Develop an alternative solution</li> <li>Synthesize information within one data set</li> </ul>   | <ul style="list-style-type: none"> <li>Synthesize information across multiple sources or data sets</li> <li>Design a model to inform and solve a practical or abstract situation.</li> </ul>   |

# Depth of Knowledge (DOK) Levels



| Level One Activities  | Level Two Activities  | Level Three Activities   | Level Four Activities   |
|---|---|--|---|
| <p>Recall elements and details of story structure, such as sequence of events, character, plot and setting.</p> <p>Conduct basic mathematical calculations.</p> <p>Label locations on a map.</p> <p>Represent in words or diagrams a scientific concept or relationship.</p> <p>Perform routine procedures like measuring length or using punctuation marks correctly.</p> <p>Describe the features of a place or people.</p> | <p>Identify and summarize the major events in a narrative.</p> <p>Use context cues to identify the meaning of unfamiliar words.</p> <p>Solve routine multiple-step problems.</p> <p>Describe the cause/effect of a particular event.</p> <p>Identify patterns in events or behavior.</p> <p>Formulate a routine problem given data and conditions.</p> <p>Organize, represent and interpret data.</p> | <p>Support ideas with details and examples.</p> <p>Use voice appropriate to the purpose and audience.</p> <p>Identify research questions and design investigations for a scientific problem.</p> <p>Develop a scientific model for a complex situation.</p> <p>Determine the author's purpose and describe how it affects the interpretation of a reading selection.</p> <p>Apply a concept in other contexts.</p> | <p>Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/solutions.</p> <p>Apply mathematical model to illuminate a problem or situation.</p> <p>Analyze and synthesize information from multiple sources.</p> <p>Describe and illustrate how common themes are found across texts from different cultures.</p> <p>Design a mathematical model to inform and solve a practical or abstract situation.</p> |

**Figure 4.4:**  
**Evaluation Tool for Assessment Instrument Quality**

| Assessment Indicators  | Description of Level 1   | Requirements of the Indicator Are Not Present | Limited Requirements of This Indicator Are Present | Substantially Meets the Requirements of the Indicator | Fully Achieves the Requirements of the Indicator | Description of Level 4   |
|--|--|---|--|---|--|--|
| Identification and emphasis on learning targets                  | Learning targets are unclear or absent from the assessment instrument. Too much attention is given to one target.                            | 1   | 2  | 3   | 4  | Clearly stated learning targets are on the assessment and connected to the assessment questions.                         |
| Visual presentation  | Assessment is sloppy, disorganized, and difficult to read. There is no room for teacher feedback.  | 1   | 2  | 3   | 4  | Assessment is neat, organized, easy to read, and well spaced. There is room for teacher feedback.                        |
| Time allotment   | Few students can complete the assessment in the time allowed.  | 1   | 2  | 3   | 4  | Test can be successfully completed in time allowed.  |
| Clarity of directions  | Directions are missing or unclear.   | 1   | 2  | 3   | 4  | Directions are appropriate and clear.  |
| Clear and appropriate scoring rubrics                            | Scoring rubric is either not in evidence or not appropriate for the assessment task.   | 1   | 2  | 3   | 4  | Scoring rubric is clearly stated and appropriate for each problem.   |
| Variety of assessment task formats                               | Assessment contains only one type of questioning strategy and no multiple choice. Calculator usage is not clear.                             | 1   | 2  | 3   | 4  | Test includes a variety of question types, assesses different formats, and includes calculator usage.                    |
| Question phrasing (precision)                                    | Wording is vague or misleading. Vocabulary and precision of language is problematic for student understanding.                               | 1   | 2  | 3   | 4  | Vocabulary is direct, fair, and clearly understood. Students are expected to attend to precision in responses.           |
| Balance of procedural fluency and demonstration of understanding | Test is not balanced for rigor. Emphasis is on procedural knowledge. Minimal cognitive demand for demonstration of understanding is present. | 1   | 2  | 3   | 4  | Test is balanced with product- and process-level questions. Higher-cognitive-demand and understanding tasks are present. |

## Sample Assessment to Analyze (NOT meant to be exemplary)

1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols  $>$ ,  $=$ ,  $<$ .

Name: \_\_\_\_\_ Date: \_\_\_\_\_

1. Circle all of the numbers **greater than 25**.

(2 points)

16

28

30

7

103

25

- Circle TRUE or FALSE after each number sentence.**

(1 point each)

2.  $7 < 10$       TRUE      FALSE

3.  $16 = 61$       TRUE      FALSE

4.  $48 > 52$       TRUE      FALSE

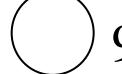
- Write  $>$ ,  $<$ , or  $=$  in each circle to compare the numbers.**

(1 point each)

5.    3  8

6.    80  40

7.    12  102

8.    91  91

9.    Vlad wrote  $56 > 39$ .      Is he correct?      YES      NO  
Show how you know.

(2 points)

## Analyze Assessments

- Which standards or learning targets are assessed?
- How are the mathematical practices assessed?
- Use the Evaluation of Assessment Tool to determine balance of DOK Levels, variety of assessment types, quality of questions and final product.
- How will the items be scored?
- What is proficiency?

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## Analyze an Assessment

- Look at the assessment.
- How does it measure against the rubric?
- How can it be improved?



What needs to be modified on  
your math assessments?

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## Contact Information

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