COSA Common Core State Standards Regional Series

"Mathematics in Action"

A Statewide Regional Series for District and School Leaders of CCSS

Secondary (6-12) Mathematics Session

Locations:

October 29, 2013 - Convention Center, Pendleton
November 5, 2013 - Holiday Inn, Wilsonville
November 8, 2013 - Deschutes Expo Center, Redmond

Mathematics Presenter:

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Track Your Progress:
Implementing the CCSSM

Shade each rectangle to show your current understanding of each learning target.

- I can design lessons focused on students learning the CCSSM.

Starting …	Getting There …	Got It!

- I can create and analyze high quality common assessments aligned to SBAC expectations.

Starting …	Getting There …	Got It!

My experience implementing the CCSSM …

What are my successes?

What are my challenges?

What Do We Expect Students To Learn?

Domains

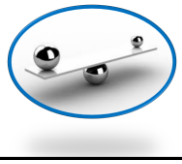
Domain	6	7	8
Ratios and Proportional Relationships (RP)	✓	✓	
The Number System (NS)	✓	✓	✓
Expressions and Equations (EE)	✓	✓	✓
Geometry (G)	✓	✓	✓
Statistics and Probability (SP)	✓	✓	✓
Functions (F)			✓

The CCSS Requires Three Shifts in Mathematics

1. **Focus:** Focus strongly where the standards focus.
2. **Coherence**: *Think* across grades, and *link* to major topics
3. **Rigor:** balance *conceptual understanding*, procedural skill and *fluency*, and *application*

Rigor: Conceptual Understanding, Fluency, and Application

- Here rigor does not mean "hard problems."
- Its a balance of three fundamental components that result in deep mathematical understanding.
- There must be variety in what students are asked to produce.



Rigor Examples

- **Conceptual Understanding**:
  - 7.RP.2d **Explain** what a point *(x, y) on the graph of a proportional relationship means in terms of the* situation, with special attention to the points (0, 0) and (1, *r) where r is the unit rate.*
- **Procedural Skill and Fluency**:
  - 7.RP.1 **Compute** unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
- **Application**:
  - 7.RP.3 **Use** proportional relationships **to solve** multistep ratio and percent **problems**. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
6. Attend to precision.

2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Where can I find tasks?

- map.mathshell.org (MARS Tasks)
- www.ccssmath.org (Resources)
- www.engageny.org/mathematics
- www.illustrativemathematics.org – click on "Illustrations"
- www.smarterbalanced.org
- www.ode.state.or.us/search/page/?id=3747 – look at Claims 2, 3 and 4
- www.insidemathematics.org
- www.teachingchannel.org

Putting it All Together

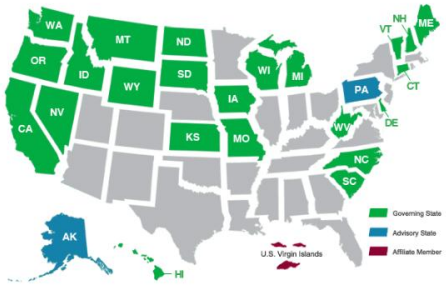
- Think of an upcoming lesson.
- What standard(s) will students learn?
- How will you know they learned it? What advancing and assessing questions will you use?
- How will you design the lesson to include tasks that incorporate the standards for mathematical practice?
- How will the lesson begin and end?

How do we know students have learned the CCSSM?

Formative vs. Summative Assessments

Formative	Summative
▪ A process during learning ▪ Descriptive feedback, use of rubrics, student self-assessment ▪ Used to support ongoing growth, improvement	▪ An event after learning ▪ Chapter tests, state assessment, end-of-year placement tests ▪ Used to measure achievement

SBAC Member States



SMARTER: Summative Multi-State Assessment Resources for Teachers and Educational Researchers
www.smarterbalanced.org

Four Claims Used in DRAFT SBAC Test Specifications

Claim #1 Concepts & Procedures — 40%
Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Claim #2 Problem Solving
Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Claim #4 Modeling & Data Analysis
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

60%

Claim #3 Communicating Reasoning
Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Table 2: Estimated testing times for Smarter Balanced Summative Assessments

Test Type	Grades	CAT	Perf Task Only	Total	In-Class Activity	Total
English Language Arts/Literacy	3-5	1:30	2:00	3:30	:30	**4:00**
	6-8	1:30	2:00	3:30	:30	**4:00**
	11	2:00	2:00	4:00	:30	**4:30**
Mathematics	3-5	1:30	1:00	2:30	:30	**3:00**
	6-8	2:00	1:00	3:00	:30	**3:30**
	11	2:00	1:30	3:30	:30	**4:00**
COMBINED	3-5	3:00	3:00	6:00	1:00	**7:00**
	6-8	3:30	3:00	6:30	1:00	**7:30**
	11	4:00	3:30	7:30	1:00	**8:30**

Analyze Assessments

- Which standards or learning targets are assessed?
- How are the mathematical practices assessed?
- Use the Evaluation of Assessment Tool to determine balance of DOK Levels, variety of assessment types, quality of questions and final product.
- How will the items be scored?
- What is proficiency?

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Content Standard Progressions 6 – 8

Number System

Similarities	Differences

Ratios and Proportional Reasoning

Similarities	Differences

Expressions and Equations

Similarities	Differences

Geometry

Similarities	Differences

Statistics and Probability

Similarities	Differences

Number System 6-8

Grade 6	Grade 7	Grade 8
Apply and extend previous understandings of multiplication and division to divide fractions by fractions. **1**. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. ***Compute fluently with multi-digit numbers and find common factors and multiples.*** **2**. Fluently divide multi-digit numbers using the standard algorithm. **3**. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **4**. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* ***Apply and extend previous understandings of numbers to the system of rational numbers.*** **5**. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6**. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	***Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*** **1**. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. **2**. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real world contexts.	***Know that there are numbers that are not rational, and approximate them by rational numbers.*** **1**. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **2**. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Number System 6-8

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **7.** Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* **8.** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **3.** Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)	

Ratios and Proportional Relationships 6-8

Grade 6	Grade 7	Grade 8
Understand ratio concepts and use ratio reasoning to solve problems.	***Analyze proportional relationships and use them to solve real-world and mathematical problems.***	*No Ratios and Proportional Relationships Domain*
1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*	
2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*	**2.** Recognize and represent proportional relationships between quantities.	
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	
a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*	c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*	
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.	
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	**3.** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	

Expressions and Equations 6-8

Grade 6	Grade 7	Grade 8
Apply and extend previous understandings of arithmetic to algebraic expressions. **1.** Write and evaluate numerical expressions involving whole-number exponents. **2.** Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.* **3.** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **4.** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	***Use properties of operations to generate equivalent expressions.*** **1.** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **2.** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* ***Solve real-life and mathematical problems using numerical and algebraic expressions and equations.*** **3.** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. **4.** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	***Work with radicals and integer exponents.*** **1.** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.* **2.** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **3.** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.* **4.** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. ***Understand the connections between proportional relationships, lines, and linear equations.*** **5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **6.** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Expressions and Equations 6-8

Reason about and solve one-variable equations and inequalities. 5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. 6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. 8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. ***Represent and analyze quantitative relationships between dependent and independent variables.*** 9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*		***Analyze and solve linear equations and pairs of simultaneous linear equations.*** **7**. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8**. Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Geometry 6-8

Grade 6	Grade 7	Grade 8
Solve real-world and mathematical problems involving area, surface area, and volume. **1**. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **2**. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **3**. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **4**. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	***Draw, construct, and describe geometrical figures and describe the relationships between them.*** **1**. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **2**. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **3**. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. ***Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.*** **4**. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **5**. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **6**. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	***Understand congruence and similarity using physical models, transparencies, or geometry software.*** **1**. Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. **2**. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **3**. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **4**. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **5**. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. ***Understand and apply the Pythagorean Theorem.*** 6. Explain a proof of the Pythagorean Theorem and its converse. 7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. ***Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.*** 9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability 6-8

Grade 6	Grade 7	Grade 8
Develop understanding of statistical variability. **1.** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **2.** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **3.** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. ***Summarize and describe distributions.*** **4.** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **5.** Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	***Use random sampling to draw inferences about a population.*** **1.** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **2.** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. ***Draw informal comparative inferences about two populations.*** **3.** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. **4.** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. ***Investigate chance processes and develop, use, and evaluate probability models.*** **5.** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **6.** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	***Investigate patterns of association in bivariate data.*** **1.** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **2.** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **3.** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **4.** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

	7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **8**. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. . *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	

Functions 6-8

Grade 6	Grade 7	Grade 8
No Functions Domain at this grade.	*No Functions Domain at this grade.*	***Define, evaluate, and compare functions.*** **1.** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.) **2.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* ***Use functions to model relationships between quantities.*** **4.** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **5.** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

7th Grade Task

7.RP

Joel and Marisa are running for president at their middle school (grades 6-8). After the votes are in, Joel and Marisa are each convinced that they have won the election:

- Joel argues that he has won a larger percentage of the overall vote than Marisa so he should be the new president.
- Marisa argues that she has won a larger percentage than Joel of the 6th grade vote and the 7th grade vote. Since the majority of the grades voted for her, she should be the new president.

Is it possible that both Joel and Marisa are making accurate claims? Explain.

Tasks taken from www.illustrativemathemtatics.org.

HS Task

A-REI, A-CED

Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it.

She said, “I wonder whether the dollar belongs inside the cash box or not.”

The price of tickets for the dance was 1 ticket for $5 (for individuals) or 2 tickets for $8 (for couples). She looked inside the cash box and found $200 and ticket stubs for the 47 students in attendance. Does the dollar belong inside the cash box or not?

Shifts in Classroom Practice

Shift 1: From *same instruction* toward *differentiated instruction.*

Same instruction for all students.	Differentiated instruction but same learning outcomes for all students.

Shift 2: From *students working individually* toward *a community of learners.*

Students work individually on tasks and seek feedback from teacher on reasonableness of strategies and solutions.	Community of learners where students hear, share, and judge reasonableness of strategies and solutions.

Shift 3: From *mathematical authority coming from the teacher or textbook* toward *mathematical authority coming from sound student reasoning.*

Correctness of solutions is determined by seeking input from teacher or textbook.	Correctness of solution is based on reasoning about the accuracy of the solution strategy.

Shift 4: From *teacher demonstrating 'how to'* toward *teacher communicating 'expectations' for learning.*

Teacher demonstrates the way in which to solve a problem and then helps students in solving it that way.	Teacher facilitates high-level performance by sharing learning goals and expectations for products that demonstrate learning.

Shift 5: From *content taught in isolation* toward *content connected to prior knowledge.*

Content presented independent of its connections to what has been previously learned.	Content presented in ways where explicit attention is given to making connections among mathematical ideas.

Shift 6: From *focus on correct answer* toward *focus on explanation and understanding*.

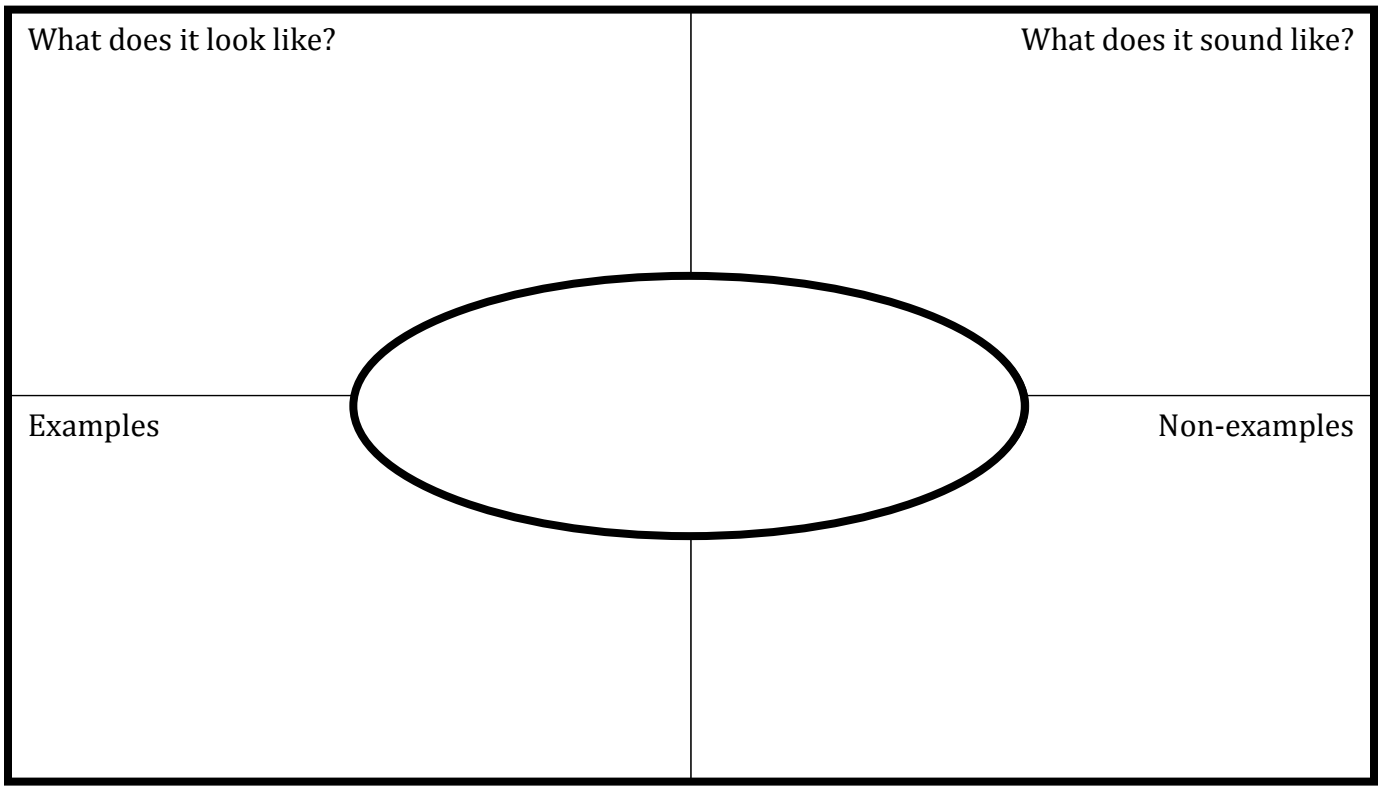
Discussions and classroom routines focus on student explanation of how they solved a task and if it is correct.	Discussions and classroom routines focus on student explanations that address why an answer is (or isn't) correct.

Shift 7: From *mathematics-made-easy for students* toward *engaging students in productive struggle.*

Mathematics is presented in small chunks and help is provided so that students reach solutions quickly and without higher level thinking.	Teacher poses tasks and challenges students to persevere and attempt multiple approaches to solving problems.

Bay-Williams, J. M., McGatha, M. B., Kobett, B. M. & Wray, J. A. *Mathematics Coaching: Resources and Tools for Coaches and Leaders, K – 12.* New York, NY: Pearson, 2013

Shifts in Classroom Practice



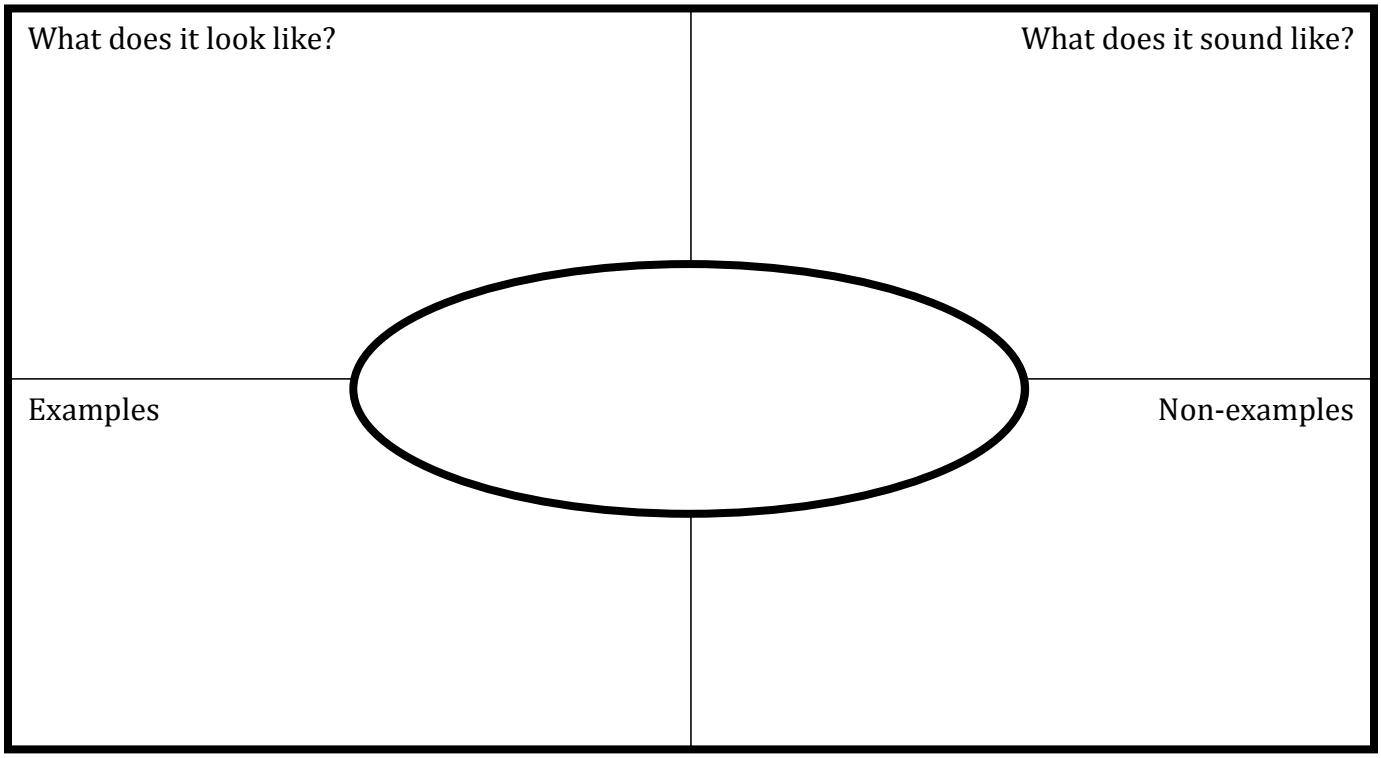


Table 2.1: Elements of an Effective Mathematics Classroom Lesson Design

	Probing Questions for Effective Lesson Design	Reflection
1. Lesson Context: Learning Targets **Procedural Fluency *and* Conceptual Understanding Balancing**	What is the learning target for the lesson? How does it connect to the bigger focus of the unit?	
	What evidence will be used to determine the level of student learning of the target?	
	Are conceptual understanding and procedural fluency appropriately balanced?	
	How will you formatively assess student conceptual understanding of the mathematics concepts *and* of the procedural skill?	
	What meaningful application or model can you use?	
	Which CCSS Mathematical Practices will be emphasized during this lesson?	
2. Lesson Process: High-Cognitive-Demand Tasks **Planning Student Discourse and Engagement**	What tasks will be used that create an a-ha student moment and leave "mathematical residue" (insights into the mathematical structure of concepts) regardless of content type at a high-cognitive-demand level?	
	How will you ensure the task is accessible to all students while still maintaining a high cognitive demand for students?	
	What strategic mathematical tools will be used during the lesson?	

	Probing Questions for Effective Lesson Design	Reflection
2. Lesson Process: High-Cognitive-Demand Tasks *(continued)*	How will each lesson *example* be presented and sequenced to build mathematical reasoning connected to prior student knowledge?	
	What are the assessing and advancing questions you might ask during guided, independent, or group practice? What are anticipated student responses to the examples or tasks?	
	How might technology and student attention to precision play a role in the student lesson experience?	
3. Introduction, Daily Review, and Closure	What activity will be used to immediately engage students at the beginning of the class period?	
	How can the daily review be used to provide brief, meaningful feedback on homework? (Five minutes maximum)	
	How will the students summarize the lesson learning targets and the key vocabulary words?	
4. Homework	How does the homework assignment provide variety and meaning to the students—including long-term review and questions—that balance procedural fluency with conceptual understanding?	

Figure 2.12:
CCSS Mathematical Practices Lesson-Planning Tool

Unit:	Date:	Lesson:
Learning target: As a result of today's class, students will be able to _____		
Formative assessment: How will students be expected to demonstrate mastery of the learning target during in-class checks for understanding?		

Probing Questions for Differentiation on Mathematical Tasks	
Assessing Questions (Create questions to scaffold instruction for students who are "stuck" during the lesson or the lesson tasks.)	**Advancing Questions** (Create questions to further learning for students who are ready to advance beyond the learning target.)

Targeted Standard for Mathematical Practice:

Which Mathematical Practice will be targeted for proficiency development during this lesson?

Tasks (Tasks can vary from lesson to lesson.)	What Will the Teacher Be Doing? (How will the teacher present and then monitor student response to the task?)	What Will the Students Be Doing? (How will students be actively engaged in each part of the lesson?)
Beginning-of-Class Routines How does the warm-up activity connect to students' prior knowledge, or how is it based on analysis of homework?		

Tasks (Tasks can vary from lesson to lesson.)	**What Will the Teacher Be Doing?** (How will the teacher present and then monitor student response to the task?)	**What Will the Students Be Doing?** (How will students be actively engaged in each part of the lesson?)
Task 1 How will the students be engaged in understanding the learning target?		
Task 2 How will the task develop student sense making and reasoning?		
Task 3 How will the task require student conjectures and communication?		
Closure How will student questions and reflections be elicited in the summary of the lesson? How will students' understanding of the learning target be determined?		

Sample Grade 11 Performance Task (from www.smarterbalanced.org)

SPEEDING TICKETS

New York state wants to change its system for assigning speeding fines to drivers. The current system allows a judge to assign a fine that is within the ranges shown in Table 1.

Table 1. New York Speeding Fines

Miles per Hour over Speed Limit	Minimum Fine	Maximum Fine
1 – 10	$45	$150
11 – 30	$90	$300
31 or more	$180	$600

Some people have complained that the New York speeding fine system is not fair. The New Drivers Association (NDA) is recommending a new speeding fine system. The NDA is studying the Massachusetts system because of claims that it is fairer than the New York system.

Table 2. Massachusetts Speeding Fines

Miles per Hour over Speed Limit	Fine
1 – 10	$100 flat charge
11 or more	$100 flat charge plus $10 for each additional mph above the first 10 mph

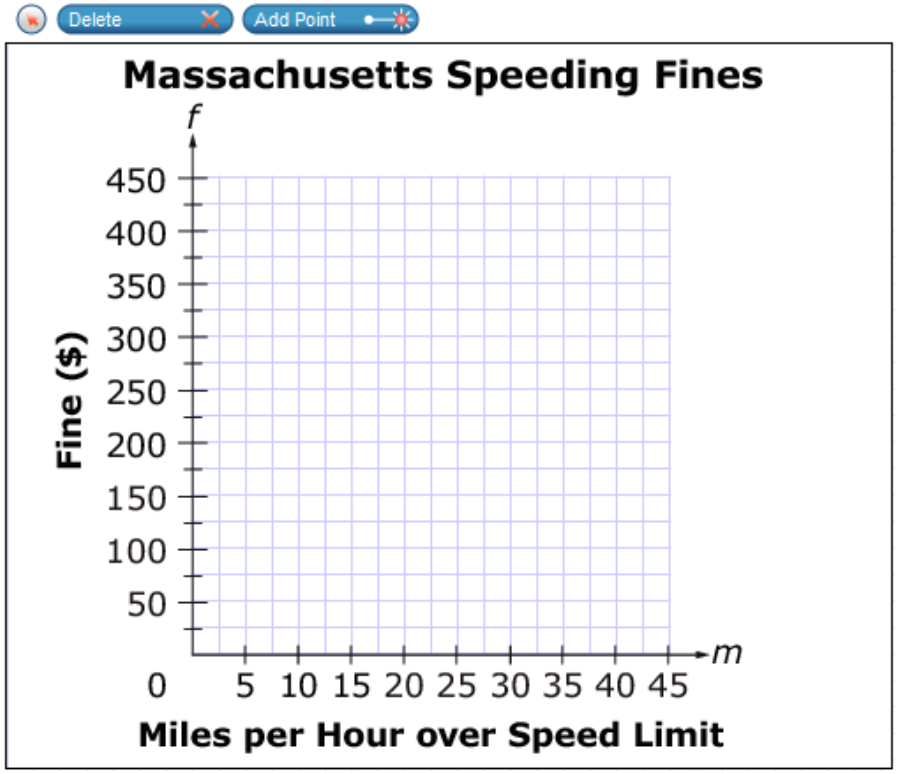
In this task, you will:

- analyze the speeding fine systems for both New York and Massachusetts.
- use data to propose a fairer speeding fine system for New York state.

1. ***Part A***

Use the information in Table 2 to plot data points for Massachusetts speeding fines.

- Plot a point to represent the fine for driving 5 mph over the speed limit.
- Plot additional points for each increment of 5 mph over the speed limit up to 45 mph over the speed limit.



Part B

Create an equation to calculate the Massachusetts speeding fine, f, based on the number of miles per hour, m, over the speed limit when $1 \leq m \leq 10$.

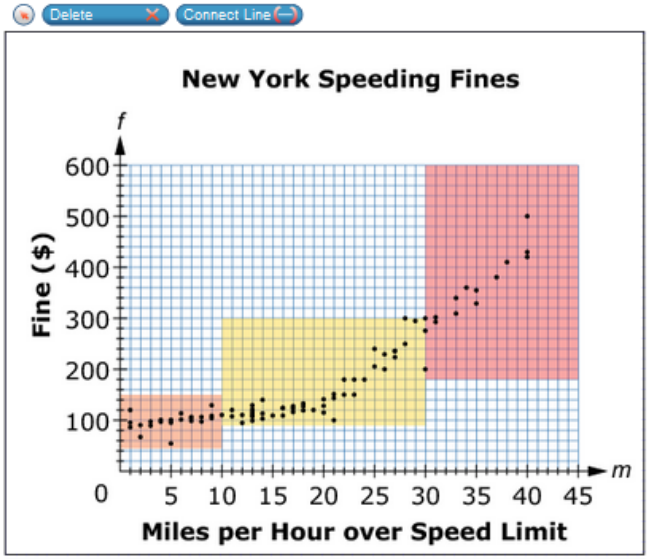
Part C

Create an equation to calculate the Massachusetts speeding fine, f, based on the number of miles per hour, m, over the speed limit when $m > 10$.

2. The graph below shows data from a sample of actual fines for driving above the speed limit in New York.

Part A

Use the Connect Line tool to create a piecewise linear model with two line segments, one for $1 \leq m \leq 20$ and one for $20 \leq m \leq 40$, that approximates the best fit for the data.



Part B

Using your model from part A, create an equation to calculate the speeding fine, f, based on the number of miles per hour, m, over the speed limit when $1 \leq m \leq 20$.

This equation will be the start of the proposed new model for the New York speeding fine system.

Part C

Using your model from part A, create an equation to calculate the speeding fine, f, based on the number of miles per hour, m, over the speed limit when $m > 20$.

This equation will complete the proposed new model for the New York speeding fine system.

3. The NDA claims that the proposed new model for the New York speeding fine system is fairer than the current system.

   Do you agree or disagree with the claim? Explain your reasoning using specific examples from this task.

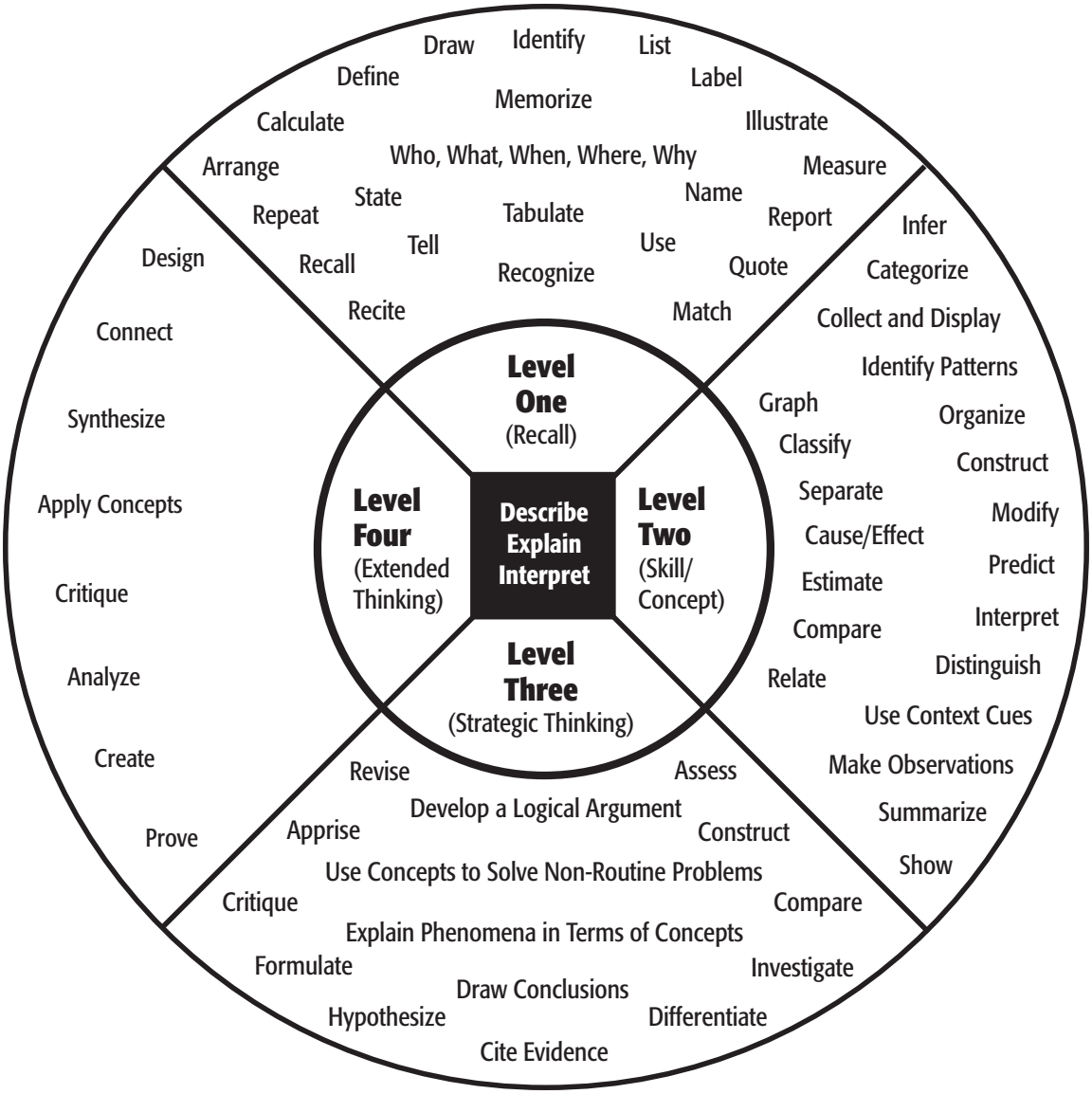
Depth of Knowledge (DOK)

Source: www.smarterbalanced.org Mathematics Content Specifications

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones & Walkup, 2009)

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	**DOK Level 1** **Recall & Reproduction**	**DOK Level 2** **Basic Skills & Concepts**	**DOK Level 3** **Strategic Thinking & Reasoning**	**DOK Level 4** **Extended Thinking**
Remember	• Recall conversations, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation.

Depth of Knowledge (DOK) Levels



Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting.	Identify and summarize the major events in a narrative.	Support ideas with details and examples.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions.
Conduct basic mathematical calculations.	Use context cues to identify the meaning of unfamiliar words.	Use voice appropriate to the purpose and audience.	Apply mathematical model to illuminate a problem or situation.
Label locations on a map.	Solve routine multiple-step problems.	Identify research questions and design investigations for a scientific problem.	Analyze and synthesize information from multiple sources.
Represent in words or diagrams a scientific concept or relationship.	Describe the cause/effect of a particular event.	Develop a scientific model for a complex situation.	Describe and illustrate how common themes are found across texts from different cultures.
Perform routine procedures like measuring length or using punctuation marks correctly.	Identify patterns in events or behavior.	Determine the author's purpose and describe how it affects the interpretation of a reading selection.	Design a mathematical model to inform and solve a practical or abstract situation.
Describe the features of a place or people.	Formulate a routine problem given data and conditions.	Apply a concept in other contexts.	
	Organize, represent and interpret data.		

Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>.

Figure 4.4:
Evaluation Tool for Assessment Instrument Quality

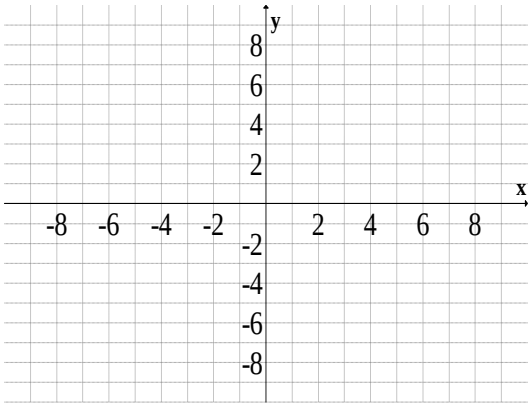
Assessment indicators	Description of Level 1	Requirements of the Indicator Are Not Present	Limited Requirements of This Indicator Are Present	Substantially Meets the Requirements of the Indicator	Fully Achieves the Requirements of the Indicator	Description of Level 4
Identification and emphasis on learning targets	Learning targets are unclear or absent from the assessment instrument. Too much attention is given to one target.	1	2	3	4	Clearly stated learning targets are on the assessment and connected to the assessment questions.
Visual presentation	Assessment is sloppy, disorganized, and difficult to read. There is no room for teacher feedback.	1	2	3	4	Assessment is neat, organized, easy to read, and well spaced. There is room for teacher feedback.
Time allotment	Few students can complete the assessment in the time allowed.	1	2	3	4	Test can be successfully completed in time allowed.
Clarity of directions	Directions are missing or unclear.	1	2	3	4	Directions are appropriate and clear.
Clear and appropriate scoring rubrics	Scoring rubric is either not in evidence or not appropriate for the assessment task.	1	2	3	4	Scoring rubric is clearly stated and appropriate for each problem.
Variety of assessment task formats	Assessment contains only one type of questioning strategy and no multiple choice. Calculator usage is not clear.	1	2	3	4	Test includes a variety of question types, assesses different formats, and includes calculator usage.
Question phrasing (precision)	Wording is vague or misleading. Vocabulary and precision of language is problematic for student understanding.	1	2	3	4	Vocabulary is direct, fair, and clearly understood. Students are expected to attend to precision in responses.
Balance of procedural fluency and demonstration of understanding	Test is not balanced for rigor. Emphasis is on procedural knowledge. Minimal cognitive demand for demonstration of understanding is present.	1	2	3	4	Test is balanced with product- and process-level questions. Higher-cognitive-demand and understanding tasks are present.

Sample Assessment to Analyze (NOT meant to be exemplary)

8.EE.8 Name: _____ Period _____

Standard: Analyze and solve linear equations and pairs of simultaneous linear equations.

1. Solve the systems of equations by graphing.



a. $y = 2x - 5$
$y = -x + 1$

b. What is the solution? _____

2. Solve the systems of equations. Show your work.
Select the correct answer.

a. $2y = 4x - 6$
$y = 3x$

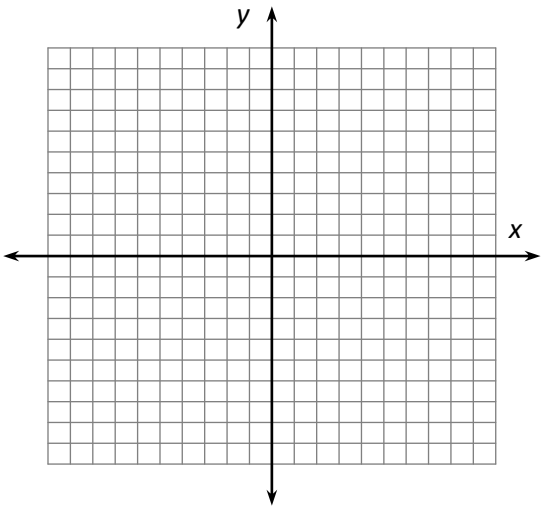
b. $4x - 2y = 12$
$-2x + 2y = 10$

3. Solve the systems of equations. Show your work.

a. $2x - 7y - 2 = 0$
$x = 4y + 24$

b. $2x + 3y = 5$
$3x + 9y = 3$

4. Water World charges $5 admission plus $.50 for each ride. The cost of Jet World is $2.75 admission plus $1.25 per ride. For what number of rides is the total cost the same at each park? What is the cost?



Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Min/Max Number of Items
				CAT	PT/ECR	SR	CR	
1. Concepts and Procedures	Priority Cluster	E. Apply and extend previous understandings of arithmetic to algebraic expressions.	1,2	p(6)=1.0	0	7	4	15/20
		F. Reason about and solve one-variable equations and inequalities.	1,2					
		A. Understand ratio concepts and use ratio reasoning to solve problems.	1,2	p(4)=1.0				
		G. Represent and analyze quantitative relationships between dependent and independent variables.	1,2	p(3)=1.0				
		B. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	1,2					
		D. Apply and extend previous understandings of numbers to the system of rational numbers.	1,2	p(2)=1.0				
	Supporting Cluster	C. Compute fluently with multi-digit numbers and find common factors and multiples.	1	p(5)=1.0	0	2	1	5/8
		H. Solve real-world and mathematical problems involving area, surface area, and volume.	2					
		I. Develop understanding of statistical variability.	1,2					
		J. Summarize and describe distributions.	1,2					

— DOK: Depth of Knowledge, consistent with the Smarter Balanced Content Specifications.
— Minimum # Scored Tasks for CAT: This column describes the minimum number of CAT items each student will receive for each target. For example, in grade 3 mathematics Claim 1 Domain 2 *Represent and interpret data,* p(1)=1.0 indicates that each student will have a 100% probability of receiving at least 1 *Represent and interpret data* CAT item.

Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Minimum Number of Items
				CAT	PT/ECR	SR	CR	
2. Problem Solving & 4. Modeling and Data Analysis	Problem Solving	A. Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.	2,3	p(3)=1.0	2	0	9	9
		B. Select and use appropriate tools strategically.	1,2					
		C. Interpret results in the context of a situation.	2					
		D. Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).	1,2,3					
	Modeling and Data Analysis	A. Apply mathematics to solve problems arising in everyday life, society, and the workplace.	2,3		4	0		
		B. Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.	2,3,4					
		C. State logical assumptions being used.	1,2					
		D. Interpret results in the context of a situation.	2,3					
		E. Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.	3,4					
		F. Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).	1,2,3					

— DOK: Depth of Knowledge, consistent with the Smarter Balanced Content Specifications.
— Minimum # Scored Tasks for CAT: This column describes the minimum number of CAT items each student will receive for each target. For example, in grade 3 mathematics Claim 1 Domain 2 *Represent and interpret data,* p(1)=1.0 indicates that each student will have a 100% probability of receiving at least 1 *Represent and interpret data* CAT item.

		G. Identify, analyze, and synthesize relevant external resources to pose or solve problems.	3,4					
Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Min/Max Number of Items
				CAT	PT/ECR	SR	CR	
3. Communicating Reasoning	n/a	A. Test propositions or conjectures with specific examples.	2	p(5)=1.0	3	0	8	8
		B. Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.	3,4					
		C. State logical assumptions being used.	2,3					
		D. Use the technique of breaking an argument into cases.	2,3					
		E. Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.	2,3,4					
		F. Base arguments on concrete referents such as objects, drawings, diagrams, and actions.	2,3					
		G. At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.)	3,4					

— DOK: Depth of Knowledge, consistent with the Smarter Balanced Content Specifications.
— Minimum # Scored Tasks for CAT: This column describes the minimum number of CAT items each student will receive for each target. For example, in grade 3 mathematics Claim 1 Domain 2 *Represent and interpret data,* p(1)=1.0 indicates that each student will have a 100% probability of receiving at least 1 *Represent and interpret data* CAT item.

Mathematics Preliminary Summative Assessment Blueprint
Target Sampling Mathematics Grade 7—Table 6b

Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Min/Max Number of Items
				CAT	PT/ECR	SR	CR	
1. Concepts and Procedures	Priority Cluster	A. Analyze proportional relationships and use them to solve real-world and mathematical problems.	1,2	p(9)=1.0	0	7	4	15/20
		D. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	1,2					
		B. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	1,2	p(6)=1.0				
		C. Use properties of operations to generate equivalent expressions.	1					
	Supporting Cluster	E. Draw, construct, and describe geometrical figures and describe the relationship between them.	2,3	p(3)=1.0	0	2	1	5/8
		F. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	1,2					
		G. Use random sampling to draw inferences about a population.	1,2	p(2)=1.0				
		H. Draw informal comparative inferences about two populations.	1,2					
		I. Investigate chance processes and develop, use, and evaluate probability models.	1,2					

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- Minimum # Scored Tasks for CAT: This column describes the minimum number of CAT items each student will receive for each target. For example, in grade 3 mathematics Claim 1 Domain 2 *Represent and interpret data,* p(1)=1.0 indicates that each student will have a 100% probability of receiving at least 1 *Represent and interpret data* CAT item.

Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Minimum Number of Items
				CAT	PT/ECR	SR	CR	
2. Problem Solving & 4. Modeling and Data Analysis	Problem Solving	A. Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.	2,3	p(3)=1.0	2	0	9	9
		B. Select and use appropriate tools strategically.	1,2					
		C. Interpret results in the context of a situation.	2					
		D. Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).	1,2,3					
	Modeling and Data Analysis	A. Apply mathematics to solve problems arising in everyday life, society, and the workplace.	2,3		4	0		
		B. Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.	2,3,4					
		C. State logical assumptions being used.	1,2					
		D. Interpret results in the context of a situation.	2,3					
		E. Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.	3,4					
		F. Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).	1,2,3					

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— Minimum # Scored Tasks for CAT: This column describes the minimum number of CAT items each student will receive for each target. For example, in grade 3 mathematics Claim 1 Domain 2 *Represent and interpret data,* p(1)=1.0 indicates that each student will have a 100% probability of receiving at least 1 *Represent and interpret data* CAT item.

		G. Identify, analyze, and synthesize relevant external resources to pose or solve problems.	3,4					
Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Minimum Number of Items
				CAT	PT/ECR	SR	CR	
3. Communicating Reasoning	n/a	A. Test propositions or conjectures with specific examples.	2	p(5)=1.0	3	0	8	8
		B. Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.	3,4					
		C. State logical assumptions being used.	2,3					
		D. Use the technique of breaking an argument into cases.	2,3					
		E. Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.	2,3,4					
		F. Base arguments on concrete referents such as objects, drawings, diagrams, and actions.	2,3					
		G. At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.)	3,4					

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— Minimum # Scored Tasks for CAT: This column describes the minimum number of CAT items each student will receive for each target. For example, in grade 3 mathematics Claim 1 Domain 2 *Represent and interpret data,* p(1)=1.0 indicates that each student will have a 100% probability of receiving at least 1 *Represent and interpret data* CAT item.

Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Min/Max Number of Items
				CAT	PT/ECR	SR	CR	
1. Concepts and Procedures	Priority Cluster	C. Understand the connections between proportional relationships, lines, and linear equations.	2	p(6)=1.0	0	7	4	15/20
		D. Analyze and solve linear equations and pairs of simultaneous linear equations.	2					
		B. Work with radicals and integer exponents.	1	p(6)=1.0				
		E. Define, evaluate, and compare functions.	1,2					
		G. Understand congruence and similarity using physical models, transparencies, or geometry software.	2					
		F. Use functions to model relationships between quantities.	1,2	p(3)=1.0				
		H. Understand and apply the Pythagorean Theorem.	2					
	Supporting Cluster	A. Know that there are numbers that are not rational, and approximate them by rational numbers.	1	p(5)=1.0	0	2	1	5/8
		I. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	2					
		J. Investigate patterns of association in bivariate data.	1,2					

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Mathematics Preliminary Summative Assessment Blueprint
Target Sampling Mathematics Grade 8—Table 6b

Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Minimum Number of Items
				CAT	PT/ECR	SR	CR	
2. Problem Solving & 4. Modeling and Data Analysis	Problem Solving	A. Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.	2,3	p(3)=1.0	2	0	9	9
		B. Select and use appropriate tools strategically.	1,2					
		C. Interpret results in the context of a situation.	2					
		D. Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).	1,2,3					
	Modeling and Data Analysis	A. Apply mathematics to solve problems arising in everyday life, society, and the workplace.	2,3		4	0		
		B. Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.	2,3,4					
		C. State logical assumptions being used.	1,2					
		D. Interpret results in the context of a situation.	2,3					
		E. Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.	3,4					
		F. Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).	1,2,3					

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— Minimum # Scored Tasks for CAT: This column describes the minimum number of CAT items each student will receive for each target. For example, in grade 3 mathematics Claim 1 Domain 2 *Represent and interpret data,* p(1)=1.0 indicates that each student will have a 100% probability of receiving at least 1 *Represent and interpret data* CAT item.

		G. Identify, analyze, and synthesize relevant external resources to pose or solve problems.	3,4					
Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Minimum Number of Items
				CAT	PT/ECR	SR	CR	
3. Communicating Reasoning	n/a	A. Test propositions or conjectures with specific examples.	2	p(5)=1.0	3	0	8	8
		B. Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.	3,4					
		C. State logical assumptions being used.	2,3					
		D. Use the technique of breaking an argument into cases.	2,3					
		E. Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.	2,3,4					
		F. Base arguments on concrete referents such as objects, drawings, diagrams, and actions.	2,3					
		G. At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.)	3,4					

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Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Min/Max Number of Items
				CAT	PT/ECR	SR	CR	
1. Concepts and Procedures	Priority Cluster	D. Interpret the structure of expressions.	1	p(2)=1.0	0	7	4	15/20
		E. Write expressions in equivalent forms to solve problems.	1,2					
		F. Perform arithmetic operations on polynomials.	1	p(1)=1.0				
		G. Create equations that describe numbers or relationships.	1,2	p(5)=1.0				
		H. Understand solving equations as a process of reasoning and explain the reasoning.	1,2					
		I. Solve equations and inequalities in one variable.	1,2					
		J. Represent and solve equations and inequalities graphically.	1,2	p(2)=1.0				
		K. Understand the concept of a function and use function notation.	1	p(1)=1.0				
		L. Interpret functions that arise in applications in terms of a context.	1,2	p(4)=1.0				
		M. Analyze functions using different representations.	1,2,3					
		N. Build a function that models a relationship between two quantities.	1,2					
	Supporting Cluster	O. Prove geometric theorems.	2	p(2)=1.0	0	2	1	5/8
		P. Summarize, represent and interpret data on a single count or measurement variable.	2	p(1)=1.0				
		A. Extend the properties of exponents to rational exponents.	1,2	p(1)=1.0				
		B. Use properties of rational and irrational numbers.	1,2					

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		C. Reason quantitatively and use units to solve problems.	1,2	p(1)=1.0				
Claim	**Content Category**	**Assessment Targets**	**DOK**	**Minimum # Scored Tasks**		**Minimum # Items per Item Type**		**Minimum Number of Items**
				CAT	**PT/ECR**	**SR**	**CR**	
2. Problem Solving & 4. Modeling and Data Analysis	Problem Solving	A. Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.	2,3	p(3)=1.0	2	0	4	5
		B. Select and use appropriate tools strategically.	1,2					
		C. Interpret results in the context of a situation.	2					
		D. Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).	1,2,3					
	Modeling and Data Analysis	A. Apply mathematics to solve problems arising in everyday life, society, and the workplace.	2,3	p(2)=1.0	p(4)=1.0	0	2	6
		B. Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.	2,3,4					
		C. State logical assumptions being used.	1,2					
		D. Interpret results in the context of a situation.	2,3					
		E. Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.	3,4					
		F. Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).	1,2,3					

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		G. Identify, analyze, and synthesize relevant external resources to pose or solve problems.	3,4					
Claim	**Content Category**	**Assessment Targets**	**DOK**	**Minimum # Scored Tasks**		**Minimum # Items per Item Type**		**Minimum Number of Items**
				CAT	**PT/ECR**	**SR**	**CR**	
3. Communicating Reasoning	n/a	A. Test propositions or conjectures with specific examples.	2	p(8)=1.0		0	8	8
		B. Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.	3,4					
		C. State logical assumptions being used.	2,3					
		D. Use the technique of breaking an argument into cases.	2,3					
		E. Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.	2,3,4					
		F. Base arguments on concrete referents such as objects, drawings, diagrams, and actions.	2,3					
		G. At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.)	3,4					

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