Principles to Actions: Ensuring Mathematical Success for All

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CCSSM Student Math Practices

1. Making sense of problems and persevering in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Looking for and expressing regularity in repeated reasoning

Smarter Balanced Math Claims

Claim 1 – Concepts and Procedures
Apply, Explain and Interpret with Precision and Fluency

Claim 2 – Problem Solving
Solve Complex Problems

Claim 3 – Communicating Reasoning
Construct Viable Arguments and Critique the Reasoning of Others

Claim 4 – Modeling and Data Analysis
Analyze Real World Scenarios and Construct Mathematical Models to Interpret and Solve Problems
NCTM, Principles to Action (2014)

- Guiding Principles for School Mathematics
  - Teaching and Learning
  - Access and Equity
  - Curriculum
  - Tools and Technology
  - Assessment
  - Professionalism

- Each principle includes evidence based:
  - Obstacles
  - Productive beliefs
  - Illustration
  - Overcoming obstacles
  - Moving to action

Effective Math Teaching Practices
NCTM, Principles to Action (2014)

1. Establish mathematics goals to focus learning
2. Implement tasks that promote reasoning and problem solving
3. Use and connect mathematical representations
4. Facilitate meaningful discourse
5. Pose purposeful questions
6. Build fluency from conceptual understanding
7. Support productive struggle in learning mathematics
8. Elicit and use evidence of student thinking

Overview of the Eight Effective Math Teaching and Learning Practices
1. Establish mathematics goals to focus learning

- Learning progressions or trajectories describe how students make transitions from prior knowledge to more sophisticated understandings.
- Both teachers and students need to be able to answer these crucial questions:
  - What mathematics is being learned?
  - Why is this important?
  - How does it relate to what has already been learned?
  - Where are these mathematical ideas going?
- Situating learning goals within the mathematical landscape supports opportunities to:
  - Build explicit connections
  - See how ideas build and relate to one another
  - Develop a coherent and connected view of the discipline.


2. Implement tasks that promote reasoning and problem solving

- Effective math teaching and learning uses carefully selected tasks as one way to motivate student learning and build new knowledge.
- Research on math tasks over the past two decades has found:
  - Not all tasks provide the same opportunities for student thinking and learning. (Stigler & Hiebert, 1999; Stigler et al., 2009)
  - Student learning is the greatest in classrooms where tasks consistently encourage high-level student thinking and the least in classrooms where tasks are routinely procedural in nature. (Stein & Gale, 1998; Stein et al., 2009)
  - Tasks with high cognitive demands are the most difficult to implement well and are often transformed into less demanding tasks. (Stein et al., 1996; Stigler & Hiebert, 1999)
- To ensure that students have the opportunity to engage in high-level thinking, teachers must regularly select and implement tasks the promote reasoning and problem solving.

[NCTM, Principles to Action (2014), p. 17]

3. Use and connect mathematical representations

- Effective mathematics teaching includes a strong focus on using varied mathematical representations.
- Using a variety of representations helps students examine a concept through more than one lens. Selected representations could include:
  - Visual representations
  - Physical representations
  - Symbolic representations
  - Contextual representations
  - Verbal representations
- When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving skills. (Kaput, Rhoades, & Silverman, 2001; Lesh, Post, and Behr, 1987)

4. Facilitate meaningful discourse

- Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class.
- Smith and Stein (2011) describe five practices for effectively using student responses in class discussions:
  - Anticipating student responses prior to the lesson
  - Monitoring students’ work on engagement with tasks
  - Selecting particular students to present their mathematical work
  - Sequencing students’ responses in specific order for discussion
  - Connecting different students’ responses and connecting responses to key mathematical ideas
- Students must have opportunities to talk with, respond to, and question one another as part of the discourse community, in ways that support the mathematics learning for all students in class


5. Pose purposeful questions

- Effective mathematics teaching relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful discourse.
- Commonalities exist across a number of questioning frameworks. Key cross cutting aspects of a number of frameworks that are particularly important within mathematics instruction include:
  - Gathering information
    - Students recall facts, definitions, or procedures
  - Probing thinking
    - Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completing of a task
  - Making the mathematics visible
    - Students discuss mathematical structures and make connections among mathematical ideas and relationships
  - Encouraging reflection and justification
    - Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work

[NCTM, Principles to Action (2014), p. 35-37]

6. Build fluency from conceptual understanding

- Effective mathematics teaching focuses on the development of both conceptual understanding and procedural fluency.
- Both NCTM and CCSS-M emphasize that procedural fluency follows and builds on a foundation of conceptual understanding, strategic reasoning, and problem solving.
- Students who use math effectively do much more than carry out procedures. Such students must also know:
  - Which procedure is appropriate and most productive for a given situation,
  - What a given procedure accomplishes, and
  - What kind of results to expect
- “Mechanical execution of procedures without understanding their conceptual basis often leads to bizarre results” (Martin, 2009, p.165)

7. Support productive struggle in learning mathematics

- Effective mathematics instruction supports students in struggling productively as they learn mathematics.
- Teacher actions to support students in productive struggle include:
  - Students engage in problems that take time to solve
    - Teachers select tasks that promote reasoning and problem solving, explicitly encouraging students to persevere, finding ways to support students without removing challenges in a task.
  - Students explain and discuss how they thought about and solved tasks
    - Teachers ask students to explain and justify how they solved a task, and value the quality of the explanation as much as the final solution.
  - Students have a responsibility and obligation to make sense of the math
    - Teachers give students the opportunity to discuss and determine the validity and appropriateness of strategies and solutions.
  - Students use important tools in making sense of the task
    - Teachers give students access to tools that will support their thinking process.
  - Students communicate one’s thinking during a task
    - Teachers ask students to explain their thinking and pose questions based on students’ reasoning, rather than on the way the teacher thinks about the task.

8. Elicit and use evidence of student thinking

- Effective mathematics teaching elicits evidence of student’s current mathematical understanding and uses it as the basis for making instructional decisions.
- A focus on evidence includes:
  - Identifying indicators of what is important to notice in students’ mathematical thinking
  - Planning for ways to elicit that information
  - Interpreting what the evidence means with respect to students’ learning
  - Deciding how to respond on the basis of students’ understanding

- Using assessment for learning means that:
  - Students are revealing their mathematical understanding, reasoning, and methods in classroom discourse and written work.
  - Students reflect on mistakes and misconceptions to improve their understanding
  - Students ask questions, responding to, and giving suggestions to support the learning of their classmates.
  - Students assess and monitor their own progress towards math learning goals, and identify areas they can improve.

Questions?

Any math teaching practices that you would like to go back and clarify?

NCTM Effective Mathematics Teaching and Learning Practices
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#2: Implement tasks that promote reasoning and problem solving

- Dan Meyer: “Math Class Needs a Makeover”

- What are characteristics of a task that places:
  - A low-level cognitive demand on students?
  - A high-level cognitive demand on students?

- What does it mean for students to be “patient” or “impatient” problem solvers?
  - How can task selection and implementation condition students to be one of these types of problem solvers?
#2: Implement tasks that promote reasoning and problem solving

• What do the following “layers of a task” refer to?
  – Visual
  – Question
  – Mathematical structure
  – Substeps

• How can an understanding of these layers help increase the cognitive demand of low-level tasks?

Layers of a math problem

NCTM Effective Teaching Practice #5: Pose purposeful questions
#5: Pose Purposeful Questions

- Jo Boaler: “How to Learn Math”

Q: How does making a simple change to the rectangle question change the nature of student thinking?

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Q: Compare the types of thinking required for the following questions:
- How do you see the shape growing?
- How many cubes are in the 100th case?

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#5: Pose Purposeful Questions

Q: How can you tell the type of question that was asked by looking at student responses?

Four key types of questions in math class:

- Gathering information: Students recall facts, definitions, or procedures.
- Probing thinking: Students explain, elaborate, or clarify their thinking, including articulating the steps in solving a problem or the logic governing the solution.
- Making the mathematics visible: Students discuss mathematical structures and make connections among mathematical ideas and reasoning.
- Encouraging reflection and justification: Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.

Practice Problem: Verrrry Interesting!

- Read the sample problem: Verrrry Interesting
- Identify “layers” within the problem (or layers that could possibly be added):
  - Visual
  - Question
  - Mathematical structure
  - Substeps
- Revise the problem in a way to promote reasoning and problem solving
- What types of questions could we ask students in the following categories?
  - Gathering information
  - Probing thinking
  - Making the mathematics visible
  - Encouraging reflection and justification
Reflection

What did Dan Meyer mean when he said:

- “The mathematics serves the conversation, the conversation doesn’t serve the mathematics.”

3-2-1 reflection:
• What are 3 big ideas that you have taken away from this discussion thus far?
• What are 2 questions that you will continue to ponder after you leave today?
• What is 1 action that you will take immediately following this day?

Sample of open source collections of tasks

- Dan Meyer 3-Act Tasks
- Dan Meyer Blog
- Desmos Class Activities
- Math Assessment Project
- Illustrative Mathematics
- Inside Mathematics
- CC Better Lesson
- Coming Soon:
  - Jo Boaler: YouCubed
- Also from NCTM:
  - Illuminations
  - Principles to Action Reflection Guide

Coming Soon

- ODE supported facilitated Independent Adoption
  - June 23-26, Southern Oregon
  - July 14-17, Hillsboro School District
    • Draft Oregon Materials Evaluation Toolkit can be found within the session materials
- Northwest Mathematics Conference
  - October 9-11, Red Lion (Portland, Jantzen Beach)
  - Jo Boaler, Dan Meyer, & more!
  - Check out the NWMC table today!