Using a Publication as a Professional Development Experience

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Effective Teaching and Learning

Mathematics Teaching Practice: Establish mathematics goals to focus learning.

I. Tasks and Questions for Reflection:

- Work together with your team to provide an example of a goal statement of the type described (p. 12–14, Discussion).
- How can the development of specific math goals support other practices for effective teaching and learning? (p. 12–14, Discussion)
- In figure 2 (pp. 14–15), Mrs. Burke says that she wants students to "better understand these different types of word problems and be able to solve them." Find solutions for each of the three problems in the figure. What equations could be written to solve each problem? Which equations match the story situation? Discuss how these three problems offer different ways to think about subtraction.
- Review the "Beliefs about teaching and learning mathematics" chart (p. 11, Obstacles). What productive beliefs are demonstrated in the conversation in figure 2? How do those beliefs move the planning forward?

II. Connections to Other Mathematics Teaching Practices:

Implement tasks that promote reasoning and problem solving.

- Describe teacher actions that would support or undercut reasoning and problem solving in the case of the three math problems in figure 2.

Use and connect mathematical representations.

- Solve the three problems in figure 2 in more than one way. Compare your strategies with those of others. Discuss how they are related.

Pose purposeful questions.

- Review the teachers' mathematical goal for the lesson in figure 2. What questions can the teachers plan to ask students during the lesson to advance them toward this goal?

III. Application to practice:

- Observe or record a mathematics lesson. Use the "Teacher and student actions" chart (p. 16) to evaluate how the lesson applies the Mathematical Teaching Practice *Establish mathematics goals to focus learning*. What evidence do you see of the teacher and student actions identified in the chart? Where do you see missed opportunities for these teacher and student actions? Give specific examples of evidence of this Mathematics Teaching Practice and ways to enhance the practice in future lessons.

Mathematics Teaching Practice: Implement tasks that promote reasoning and problem solving.

I. Tasks and Questions for Reflection:

- Work individually or with a partner to solve task A: Smartphone Plans in figure 5 (p. 20). After explaining your decision about which phone plan is better, solve the problem in a different way. Share solutions with your team and discuss the different strategies used, the multiple entry points that the problem offers, and the different ways that the task promotes reasoning. Compare task A with task B in these respects.
- What prior knowledge or experience would students need to solve the tasks in Figures 5, 6, and 7?
- What are the characteristics of a task that places a high-level cognitive demand on students?
- How could you take a low-level task and increase its cognitive demand? (Consider rewriting a sample task or textbook problem.)
- What types of questions could you ask, or what types of moves could you make, to support students who struggle to get started on a problem-solving task, without diminishing the cognitive demand of that task?
- Review the "Beliefs about teaching and learning mathematics" chart (p. 11, Obstacles) and examine figure 8 (p. 23), which presents two algebra classrooms using the Smartphone Plans task. What beliefs are evident in Ms. Carson's and Ms. McDonald's classrooms? What impact do those beliefs have on students' opportunities for reasoning and problem solving in the lesson?

II. Connections to Other Mathematics Teaching Practices:

Establish mathematical goals to focus learning.

- Consider the problem in figure 7. Identify a mathematical goal that this problem might support.

Use and connect mathematical representations.

- Review the Smartphone Plans task (task A in figure 4) or the task on number pairs that make 10 (figure 7). What representations could students use to solve the problem? Show how students might use different representations in solving. Discuss the relationships among all the representations generated for the problem.

Pose purposeful questions.

- Review the synopsis of Ms. McDonald's classroom in figure 8. What questions does Ms. McDonald ask her students and why? How do these questions engage and challenge her students?

III. Application to practice:

- Observe or record a mathematics lesson. Use the "Teacher and student actions" chart (p. 24) to evaluate how the lesson applies the Mathematical Teaching Practice *Implement tasks that promote reasoning and problem solving*. What evidence do you see of the teacher and student actions listed in the chart? Where do you see missed opportunities for these teacher and student actions? Give specific examples of evidence of this Mathematics Teaching Practice and ways to enhance the practice in future lessons.

Mathematics Teaching Practice: Use and connect mathematical representations.

I. Tasks and Questions for Reflection:

- Revisit the three problems for second-grade students in figure 2 (pp. 14–15). Show how students might solve each problem by using different representations. Discuss the relationships among all the representations generated for each problem.
- Look at the parenthetical example on p. 26 about describing a real-world situation for 3 x 29 or $y = 3x + 5$. Use contexts or representations to show how the expression 3 x 29 is related to the equation $y = 3x + 5$.
- Review the "Beliefs about teaching and learning mathematics" chart (p. 11, Obstacles). What productive beliefs are evident in the Mr. Harris's classroom, shown in figure 10 (pp. 27–28)? How do those beliefs support students in making connections among different representations of the problem?
- Analyze samples of student work from a lesson that you have taught this year. Find examples in which students have used different representations to solve the same problem. Make a plan to connect those representations explicitly in future lessons. Find relationships between and among the representations and think about how you could use the students' work to develop their understanding of a concept.

II. Application to practice:

- Observe or record a mathematics lesson. Use the "Teacher and student actions" chart (p. 16) to evaluate how the lesson applies the Mathematical Teaching Practice *Use and connect mathematical representations*. What evidence do you see of the teacher and student actions identified in the chart? Where do you see missed opportunities for these teacher and student actions? Give specific examples of evidence of this Mathematics Teaching Practice and ways to enhance the practice in future lessons.

Mathematics Teaching Practice: Facilitate meaningful mathematical discourse.

I. Tasks and Questions for Reflection:

- Simply having students talk does not necessarily advance the mathematical goals of a lesson. How can the five practices identified on page 30, as described by Smith and Stein (2011), support and facilitate the purposeful exchange of ideas in the mathematics classroom?
- Review the Candy Jar task in figure 12 (p. 31) and the conversation from Mr. Donnelly's implementation of the Candy Jar task, shown in figure 13 (pp. 33–34). What do the authors mean when they say, "Mr. Donnelly facilitates rather than directs" this discussion (p. 34)? Give specific examples.
- Review the "Beliefs about teaching and learning mathematics" chart (p. 11, Obstacles). What productive beliefs are evident in the Mr. Donnelly's classroom (see fig. 13)? What impact do those beliefs have on the classroom discourse?
- Discuss how the use of different talk structures (whole-class, small-group, teacher-led, student-led, etc.) can affect mathematical discourse in the classroom.

II. Connections to Other Mathematics Teaching Practices:

Establish mathematical goals to focus learning.

- Review the illustration of Mr. Donnelly's seventh-grade class, shown in figure 13 (p. 33–34). What is the mathematical goal of this task? Discuss how Mr. Donnelly's actions move the class toward this goal.

Elicit and use evidence of student thinking.

- Discuss how the purposeful exchange of ideas in the mathematics classroom can create opportunities to elicit and use evidence of student thinking.

III. Application to practice:

- Observe or record a mathematics lesson. Use the "Teacher and student actions" chart (p. 35) to evaluate how the lesson applies the Mathematical Teaching Practice *Facilitate meaningful mathematical discourse*. What evidence do you see of the teacher and student actions identified in the chart? Where do you see missed opportunities for these teacher and student actions? Give specific examples of evidence of this Mathematics Teaching Practice and ways to enhance the practice in future lessons.

Mathematics Teaching Practice: Pose purposeful questions.

I. Tasks and Questions for Reflection:

- Teachers use a variety of questions in their instruction (see fig. 14, pp. 36–37), including questions that should elicit mathematical reasoning and justification. Unfortunately, teachers too often employ these questions in a "funneling" manner (see fig. 16, pp. 39–40). Brainstorm with your team to identify barriers that might prevent teachers from moving from "funneling" to "focusing" questions.
- Learning involves a cognitive reorganization of individual beliefs. This reorganization demands some degree of dissonance. How do funneling questions discourage dissonance and how do focusing questions encourage dissonance?
- In questioning small groups of students working on a problem, a teacher noticed that when she asked a "focusing" question, the students continued to look at their work and continued to engage in their own dialogue. When she asked a "funneling" question, the students looked up at the teacher. Comment on these observations.
- Identify a math task that you might give to your students. State the learning goal, and then use the task to create a list of related questions using the framework in figure 14 (pp. 36–37). It will be helpful to first *anticipate* likely student responses and misconceptions (see Smith & Stein's practice 1, p. 30).
- If your district uses a specific framework for questioning (e.g., Bloom's Taxonomy or Webb's Depth of Knowledge), compare that framework with the framework of types of questions shown in figure 14 (pp. 36–37). Discuss any connections.
- Review the patterns of questions about the Coin Circulation task posed in each classroom, as shown in figure 16 (pp. 38–39). Classify the question types according to the framework in figure 14. What do you notice?
- Review the "Beliefs about teaching and learning mathematics" chart (p. 11, Obstacles). What beliefs are evident in the two questioning examples shown in figure 16? What impact do those beliefs have on students' opportunities to make sense of the mathematics and advance their reasoning?

II. Connections to Other Mathematics Teaching Practices:

Establish mathematical goals to focus learning.

- Why is it essential for teachers to have a clear learning goal when facilitating discussion that focuses on student thinking?

Elicit and use evidence of student thinking.

- Review the patterns of questioning in figure 16. Students in both classrooms notice that range for the middle 50 percent of the pennies is from 3 to 19 years old, observed by student 5 (S5) in the funneling example and by student 2 (S2) in the focusing example. Compare the reactions of the two teachers to these students' observations. How might the teacher in the funneling example have responded to student 5 in a different way to gain a better understanding of what the student or other students knew and understood?

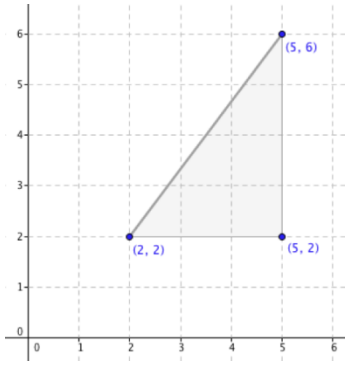
III. Application to practice:

- Observe or record a mathematics lesson. Use the framework for types of questions used in mathematics teaching (fig. 14, p. 36) to evaluate questions asked during the lesson. Tally questions by type. Provide specific examples of questions for each type. Reflect on the distribution of questioning and how the specific questions cited advanced or limited progress toward the mathematical goals of the lesson.
- Observe or record a mathematics lesson. Use the "Teacher and student actions" chart (p. 41) to evaluate how the lesson applies the Mathematical Teaching Practice *Pose purposeful questions*. What evidence do you see of the teacher and student actions identified in the chart? Where do you see missed opportunities for these teacher and student actions? Give specific examples of evidence of this Mathematics Teaching Practice and ways to enhance the practice in future lessons.

Mathematics Teaching Practice: Build procedural fluency from conceptual understanding.

I. Tasks and Questions for Reflection:

- Consider a right triangle in the first quadrant of the coordinate plane (see the example below). Label each point. Discuss with your team how the triangle's measurements are related to the distance formula (shown on p. 44). Specifically, what two x values are represented in the coordinates of your right triangle? What two y values are represented? What is the difference between the two x values? How does that relate to the measurements of the triangle? What is the difference between the two y values? How does that relate to the measurements of the triangle? How would you find the length of the triangle's hypotenuse? And so on.



- Review the different methods for multi-digit multiplication shown in figure 18 (p. 45). Discuss how the methods are interrelated. For example, the traditional algorithm gives a partial product of 368. Where can you find 368 in the other methods?
- Review the "Beliefs about teaching and learning mathematics" chart (p. 11, Obstacles). What

beliefs are evident in Mr. Donnelly's implementation of the Candy Jar task (p. 46, Illustration, see also figs. 12, 13, and 19)? What impact do those beliefs have on students' opportunities for reasoning and problem solving in the lesson?

II. Connections to Other Mathematics Teaching Practices:

Use and connect mathematical representations.

- Look at David's and Anna's work in figure 17 (p. 43). How could Anna's reasoning help David understand his mistake? What other representations could the teacher use to support the students' thinking here?

Pose purposeful questions.

- What are some common mistakes that students make when solving multi-digit multiplication problems like 68 x 46? Give examples, and discuss questions that teachers could ask students in each case. Explain the purpose of and possible responses to each question.

Elicit and use evidence of student thinking.

- Suppose that a student multiplies 68 x 46 and gets 2448. What error has the student made? How could a teacher use one or more of the methods shown in figure 18 to help this student?

III. Application to practice:

- Observe or record a mathematics lesson. Use the "Teacher and student actions" chart (pp. 47–48) to evaluate how the lesson applies the Mathematical Teaching Practice *Build procedural fluency from conceptual understanding*. What evidence do you see of the teacher and student actions identified in the chart? Where do you see missed opportunities for these teacher and student actions? Give specific examples of evidence of this Mathematics Teaching Practice and ways to enhance the practice in future lessons.

Mathematics Teaching Practice: Support productive struggle in learning mathematics.

I. Tasks and Questions for Reflection:

- Review the problem-solving strategies suggested by Ms. Ramirez's students in figure 21 (p. 51). Solve the problem by using each of these student-suggested strategies.
- Review the video "My Favorite No: Learning From Mistakes" (https://www.teachingchannel.org/videos/class-warm-up-routine). Choose a common student error and create a "favorite no" for the problem presented in figure 21. Why is this common error useful to know?
- Review the "Beliefs about teaching and learning mathematics" chart (p. 11, Obstacles). What beliefs are evident in Ms. Flahive's and Ms. Ramirez's classrooms (see fig. 21)? What impact do those beliefs have on students' opportunities to grapple with the mathematical ideas and relationships in the problem?

II. Connections to Other Mathematics Teaching Practices:

Use and connect mathematical representations.

- Read the problem presented in figure 21. Show how students might solve the problem by using different representations. Discuss the relationships among all the representations generated for the problem.

Pose purposeful questions.

- Consider the students' strategies in Ms. Ramirez's class (fig. 21), and give examples of questions that teachers might pose to facilitate their reasoning and perseverance.

III. Application to practice:
- Observe or record a mathematics lesson. Figure 20 (p. 49) presents a chart adapted from Smith (2000), which redefines student and teacher success in relation to teachers' expectations and actions and indicators of success. Use the chart to evaluate teacher and student attitudes that are evident in the lesson. Provide specific examples. Reflect on how teacher and student attitudes advanced or limited progress toward the mathematical goals of the lesson.
- Observe or record a mathematics lesson. Use the "Teacher and student actions" chart (p. 52) to evaluate how the lesson applies the Mathematical Teaching Practice *Support productive struggle in learning mathematics*. What evidence do you see of the teacher and student actions identified in the chart? Where do you see missed opportunities for these teacher and student actions? Give specific examples of evidence of Mathematics Teaching Practice and ways to enhance the practice in future lessons.

Mathematics Teaching Practice: Elicit and use evidence of student thinking.

I. Tasks and Questions for Reflection:
- Discuss how other mathematical teaching practices, such as selecting tasks, facilitating discourse, and posing questions, are connected to this practice.
- Review the "Beliefs about teaching and learning mathematics" chart (p. 11, Obstacles). What beliefs are evident in Ms. Lewis's classroom in figure 22 (pp. 55–56)? What impact do those beliefs have on the teacher's ability assess her students' understanding and make appropriate adjustments to her instruction?

II. Connections to Other Mathematics Teaching Practices:

Use and connect mathematical representations.
- Look at Maddie's work and Gabe's work, shown in figure 22. How could the teacher leverage the students' representations to develop Maddie's understanding of the problem?

Build procedural fluency from conceptual understanding.
- Look again at Maddie's work and Gabe's work, shown in figure 22. What conceptual understandings do the children demonstrate? What understanding does Maddie appear to be missing?

III. Application to practice:
- Observe or record a mathematics lesson. Use the "Teacher and student actions" chart (p. 56) to evaluate how the lesson applies the Mathematical Teaching Practice *Elicit and use evidence of student thinking*. What evidence do you see of the teacher and student actions identified in the chart? Where do you see missed opportunities for these teacher and student actions? Give specific examples of evidence of this Mathematics Teaching Practice and ways to enhance the practice in future lessons.

Essential Elements

Access and Equity

- Read the full statement of the Access and Equity Principle (the three blue lines in italics on p. 59) three times: first, aloud at your table, second, silently to yourself, and finally, aloud to the whole group.
- With a partner, and in your own words, write a sentence that conveys the same ideas as the full statement of the Access and Equity Principle (the three blue lines in italics on p. 59).
- Referring to the full statement of the Access and Equity Principle, explain how *high-quality* mathematics differs from *advanced* mathematics?
- Describe how "high-quality" differs from advanced mathematics?
- Generate a list of topics typically taught in Advance Mathematics that all students should access. Dig into this idea of access by using the topic, Exponential Growth. Give several reasons why every student needs to know something about Exponential Growth. Continue by making a list of concepts dealing with exponential growth that ALL, MOST and FEW students need to know. Develop 3 tiers of activities that would allow ALL students "access" to exponential growth.
- What are the biggest obstacles that you face in ensuring access and equity for all students? The authors note that, in many classrooms, the Mathematical Teaching Practices described in this document are inconsistently or ineffectively implemented (p. 61). Discuss how specific changes in teaching practices can help to overcome the obstacles you identified.
- Looking at the "Beliefs about access and equity in mathematics" chart (pp. 63–64), can you find an unproductive belief and a productive belief that you can relate to? Be prepared to share your thoughts with your team.
- Briefly discuss why *Principles to Actions* places Access and Equity as the first Essential Element?

Curriculum

- If the Common Core State Standards drive the content that you teach or will teach, what resources would you like and need to help ALL students meet those standards? How would these resources affect your teaching practices?
- What needs to be in a curriculum to guarantee that students develop fluency with the processes and practices identified in the Standards for Mathematical Practice in the Common Core State Standards?
- What would you need to change in your personal style of teaching mathematics to overcome some of the obstacles to achieving the Curriculum Principle?
- With a partner, outline steps or actions that you intend to take to reshape your curriculum for a closer match with the vision captured in the full statement of the Curriculum Principle (three blue lines in italics on p. 70)? Pick one action that you will DO, and share how you will know you have reached that goal.

Tools and Technology

- The opening sentence on the Tools and Technology Principal states, "For meaningful learning of mathematics, tools and technology must be indispensable features of the classroom" (p. 78). What does, or would, such a classroom look like and sound like?
- Imagine that cost is not an issue, and develop a list of personal obstacles that might still keep you from being part of the classroom that you described above?
- How could you or others overcome these obstacles? Make a list of ALL pathways for overcoming the obstacles. When you have your list, circle three pathways that you could travel down to make the classroom that you described above a reality.
- A useful question to consider is, "Are you abusing the technology or using it to make sense of mathematical ideas?" Define the difference between abusing technology and using it in sense making by writing a quick lesson on a high-rigor topic of your choice.

Assessment

- Read the "Obstacles" section (pp. 89–92) for the Assessment Principle. Underline or highlight three ideas that resonated with you, and write down one question that you have about obstacles to effective assessment. Share your ideas and question with four different people.
- With a partner, make a detailed list of actions that you or others could take to overcome some of the obstacles shared with you.

Professionalism

- Before reading the section on the Professionalism Principle, write out your personal definition of *professionalism*.
- Read the section on the Professionalism Principle (pp. 99–108), and then share some of the obstacles that you face in professional collaboration.
- Using the "Beliefs about professionalism in mathematics education" chart (pp. 102–103), engage with a partner in a friendly discussion in which one of you defends "unproductive beliefs" and the other defends "productive beliefs." Alternate positions and discuss every pair of beliefs.
- Coaching is a key to overcoming obstacles. How could coaching be (or how is it) a positive force in your school's efforts to overcome obstacles? How could it be (or how is it) a positive force in your personal growth as a teacher?
- Review your definition of *professionalism* and make revisions if you wish. Share your revised definition with your team.
- With your team, come up with at least two actions that team members believe will improve their professionalism.

Additional Reflection Guides can be found at www.nctm.org/reflectionguides.